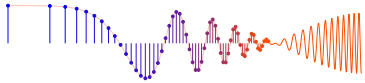

PROBLEM:

Solve the following equation for z: $z^3 = -1 + j$

Make sure you find all possible answers.

Express your answer(s) in polar form.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$$z^3 = -1 + j = \sqrt{2}\, e^{j 3\pi/4}$$

Since $e^{j 2\pi \ell} = 1$ when ℓ is an integer, we solve

$$z^3 = \sqrt{2}\, e^{j 3\pi/4} e^{j 2\pi \ell} \qquad \ell = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \quad z = 2^{1/6} e^{j \pi/4} e^{j 2\pi \ell/3} \qquad \ell = 0, 1, 2.$$

Since $e^{j 2\pi \ell/3}$ will equal 1 when $\ell = 3$, there are only 3 answers.

$2^{1/6} = 1.123$

$$z = \left\{ 2^{1/6} e^{j \pi/4},\ 2^{1/6} e^{j 11\pi/12},\ 2^{1/6} e^{j 19\pi/12} \right\}$$