PROBLEM:

An amplitude modulated (AM) cosine wave is represented by the formula

$$x(t) = [3 + \sin(\pi t)]\cos(13\pi t + \pi/2)$$

(a) Use *phasors* to show that $x(t)$ can be expressed in the form:

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2) + A_3\cos(\omega_3 t + \phi_3)$$

where $\omega_1 < \omega_2 < \omega_3$; i.e., find $A_1,\ A_2,\ A_3,\ \phi_1,\ \phi_2,\ \phi_3,\ \omega_1,\ \omega_2,\ \omega_3$ in terms of A, ω_0, and ω_c.

(b) Sketch the two-sided spectrum of this signal on a frequency axis. Be sure to label important features of the plot. Label your plot in terms of the numerical values of the A_i's ϕ_i's and ω_i's.

(c) Determine the minimum sampling rate that can be used to sample $x(t)$ without any aliasing.

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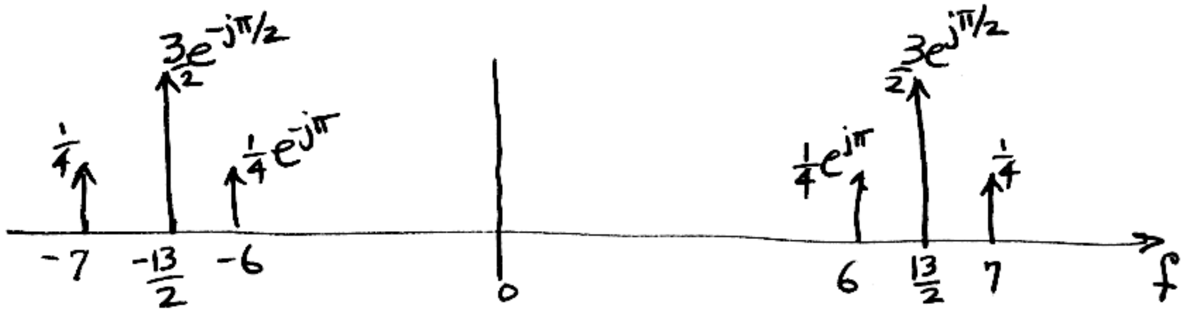
SOLUTION

(a) $$x(t) = \left[3 + \frac{1}{2j}\left(e^{j\pi t} - e^{-j\pi t} \right) \right] \left(\frac{e^{j13\pi t} e^{j\pi/2} + e^{-j13\pi t} e^{-j\pi/2}}{2} \right)$$

$$x(t) = \frac{3}{2} e^{j\pi/2} e^{j13\pi t} + \frac{3}{2} e^{-j\pi/2} e^{-j13\pi t} + \frac{1}{4} e^{j14\pi t} + \frac{1}{4} e^{-j\pi} e^{-j12\pi t}$$
$$+ \frac{1}{4} e^{j\pi} e^{j12\pi t} + \frac{1}{4} e^{-j14\pi t}$$

$$= 3\cos(13\pi t + \pi/2) + \frac{1}{2}\cos 14\pi t + \frac{1}{2}\cos(12\pi t + \pi)$$

(b)



(c) Highest Freq = 7 Hz.

$\Rightarrow$ min sampling rate $> 2 f_{HIGHEST}$

$\therefore \quad F_s > 14$ samples/sec.

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