PROBLEM:

Solve the following complex-valued equations. Reduce the answers to a simple numerical form.

(a) Find all solutions of $z^{13} = -1$. Express your answers for z in $\boxed{\text{polar}}$ form. How many *different* solutions exist?

(b) The following equation depends on n and T. Whenever T is assigned a value, the equation must then be true for all n.

$$e^{j(\pi/13)n} = e^{j13\pi nT} \qquad \text{for all } n$$

Find all possible values for T for which the equation will be true.

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SOLUTION

(a) $z^{13} = -1 = e^{j\pi} \cdot e^{j2\pi\ell}$ ← ℓ = integer.

NOTE: $-1 = e^{j\pi}$

$\Rightarrow z = e^{j(\pi + 2\pi\ell)/13}$ $\qquad \ell = 0, 1, 2, \ldots 12$

There are 13 distinct solutions:

$$\left\{ e^{j\pi/13}, e^{j3\pi/13}, e^{j5\pi/13}, e^{j7\pi/13}, \ldots e^{j\pi}, e^{j15\pi/13}, \ldots e^{j25\pi/13} \right\}.$$

(b) $e^{j(\pi/13)n} = e^{j13\pi n T}$

$e^{j\pi/13\, n} e^{j2\pi\ell n} = e^{j13\pi n T}$ $\qquad \ell$ = integer

EQUATE EXPONENTS:

$\frac{\pi}{13} n + 2\pi\ell n = 13\pi n T$ $\qquad$ ↙ CANCEL π & n.

$\frac{1}{13} + 2\ell = 13T$

$\Rightarrow T = \frac{1}{169} + \frac{2\ell}{13}$ $\qquad \ell = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots$

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