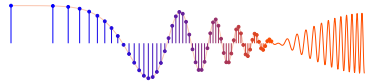

PROBLEM:

In the rotating disk and strobe demo shown in class we observed that different flashing rates of the strobe light would make the spot on the disk stand still or move in different directions.

(a) Assume that the disk is rotating counter-clockwise at a constant speed of 12 revolutions per second. If the flashing rate is 14 times per second, express the movement of the spot on the disk as a complex phasor, $p[n]$, that gives the position of the spot at the n-th flash. Assume that the spot is at the top when $n = 0$ (the first flash).

(b) For the conditions in part (a), determine the apparent speed (in revolutions per second) and direction of movement of the "strobed" spot.

(c) *Now assume that the rotation speed of the disk is unknown.* If the flashing rate is 17 times per second, and the spot on the disk moves clockwise by 20 degrees with each flash, determine the rotation speed of the disk (in rev/sec). If the answer is not unique give all possible rotation speeds.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $p(t) = e^{+j(2\pi(12)t + \varphi)}$ ← CONTINUOUS MOTION OF THE SPOT

$p[n] = p(t)\Big|_{t=n/f_s} = p(n/14)$

$p[n] = e^{+j(2\pi(12)n/14 + \varphi)}$

DIGITAL FREQ: $2\pi\left(\frac{12}{14}\right)$ ← SAME AS $-2\pi\left(\frac{1}{7}\right)$

phase at $n=0$ is $\varphi = \pi/2$.

$\therefore\ p[n] = e^{-j(2\pi(1/7)n - \pi/2)}$

(b) Convert "n" back to contin-time "t"

$n \longleftrightarrow F_s t = 14t$

$\therefore$ OBSERVED $\hat{p}(t) = p[n]\Big|_{n=14t} = e^{-j(2\pi(\frac{1}{7})14t - \pi/2)}$

$\hat{p}(t) = e^{-j(2\pi(2)t - \pi/2)}$ ← FREQ = 2 rev/sec CLOCKWISE

(c) DISK MOVES $-20°$ per flash

$\Rightarrow$ 1 rev per 18 flashes.

If $F_s = 17$ flashes/sec $\Rightarrow$ speed $= -\frac{17}{18}$ rev/sec.

However, disk could make addition revolutions:

$20° + 360°\ell$ ← per flash

$$\text{speed} = \frac{17}{360°/(-20+360\ell)} = \frac{17(-20+360\ell)}{360} = -\frac{17}{18} + 17\ell$$

$\ell = 0, \pm 1, \pm 2, \ldots$

Possible speeds:

$16\frac{1}{8}$ CCW, $33\frac{1}{8}$ CCW, ...

$17\frac{17}{18}$ CW, $34\frac{17}{18}$ CW, ...