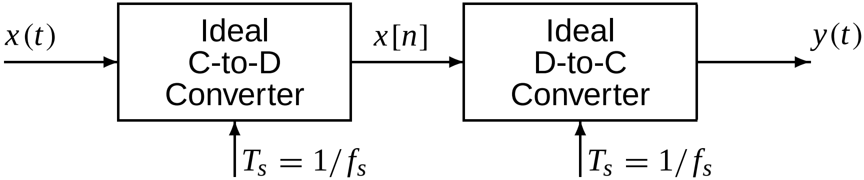
# PROBLEM:

Consider the following system.



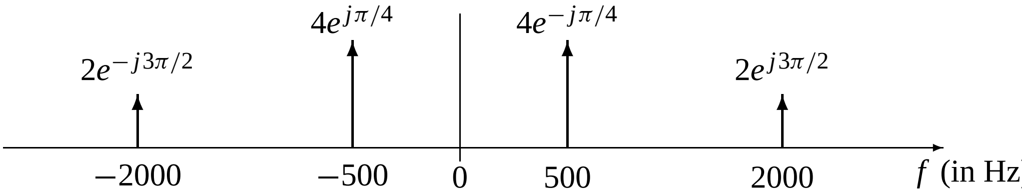

(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 10\cos(0.18\pi n + \pi/4)$$

If the sampling rate is $f_s = 2500$ samples/second, determine two *different* continuous-time signals $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have been inputs to the above system; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/2500$. Both of these input signals should have a frequency less than 2500 Hz. Give a formula for each signal.

(b) For $x[n]$ given in part (a), what is the frequency of the analog signal $y(t)$ that will be reconstructed by the ideal D-to-C converter operating at sampling rate 2500 samples/second?

(c) If the input $x(t)$ is given by the two-sided spectrum representation shown below, determine a $\boxed{\text{simple}}$ formula for $y(t)$ when $f_s = 2500$ samples/sec. (for both the C/D and D/C converters).




McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.


**SOLUTION**

(a) $x[n] = 10\cos(0.18\pi n + \pi/4)$ $\quad F_s = 2500$

$x(t) = 10\cos(2\pi F_o t + \varphi).\Big|_{t \leftarrow n/F_s}$.

$2\pi \frac{F_o}{F_s} = 0.18\pi \Rightarrow F_o = 0.09 F_s = 225\,\text{Hz}$.

This is the ordinary sampling case, so

$x(t) = 10\cos(2\pi(225)t + \pi/4)$.

The other possibility would be the "FOLDING" case.

$2\pi \frac{F_o}{F_s} = 2\pi(1-\alpha) = 2\pi(1-0.09)$

$\Rightarrow F_o = 0.91 F_s = 2250\,\text{Hz}$.

then $x(t) = 10\cos(2\pi(2250)t - \pi/4)$

Need phase reversal for folding.

(b) Construct lowest freq sinusoid.

$\therefore y(t) = 10\cos(2\pi(225)t + \pi/4)$

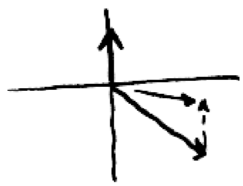
225 Hz will be freq of output.

(c) $x(t) = 8\cos(2\pi(500)t - \pi/4) + 4\cos(2\pi(2000)t + 3\pi/2)$

$x[n] = 8\cos(2\pi(\frac{1}{5})n - \pi/4) + \underbrace{4\cos(2\pi(4/5)n + 3\pi/2)}_{4\cos(2\pi(1/5)n - 3\pi/2)}$

Use PHASOR-ADD THM.

$x[n] = 5.9\cos(2\pi(1/5)n - 0.09\pi)$



$y(t) = 5.9\cos(2\pi(500)t - 0.09\pi)$

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Prentice Hall, Upper Saddle River, NJ 07458. 