## PROBLEM:

The diagram in Figure 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

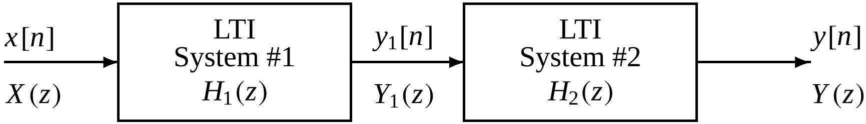


Figure 1: Cascade connection of two LTI systems.

(a) Use $z$-transforms to show that the system function for the system function of the overall system (from $x[n]$ to $y[n]$) is $H(z) = H_2(z)H_1(z)$, where $Y(z) = H(z)X(z)$.

(b) Derive a condition on $H(z)$ that guarantees that the output signal will always be equal to the input signal.

(c) Suppose that System #1 is an FIR filter described by the difference equation $y_1[n] = x[n] + \frac{5}{6}x[n-1]$ and System #2 is described by the system function $H_2(z) = 1 - 2z^{-1} + z^{-2}$. Determine the system function of the overall cascade system.

(d) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Figure 1.

(e) Plot the poles and zeros of $H(z)$ in the complex $z$-plane.

(f) If System #1 is the difference equation: $y_1[n] = x[n] + \frac{5}{6}x[n-1]$, find a system function $H_2(z)$ so that output of the cascaded system will always be equal to its input. In other words, find $H_2(z)$ which will undo the filtering action of $H_1(z)$. This is called *deconvolution*.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.


SOLUTION

(a) $\left.\begin{aligned} Y_1(z) &= H_1(z)X(z) \\ Y(z) &= H_2(z)Y_1(z) \end{aligned}\right\} \Rightarrow Y(z) = \underbrace{H_2(z)H_1(z)}_{H(z)}X(z)$

$\therefore$ CASCADE SYSTEM HAS SYSTEM FUNCTION

$$\boxed{H(z) = H_2(z)H_1(z)}$$

(b) To make $Y(z) = X(z)$ always:

$Y(z) = H(z)X(z) \quad \Longrightarrow \quad \boxed{H(z) = 1}$

(c) $H_1(z) = 1 + \frac{5}{6}z^{-1}$

$\therefore\ H(z) = (1 - 2z^{-1} + z^{-2})(1 + \frac{5}{6}z^{-1})$

$H(z) = 1 - \frac{7}{6}z^{-1} - \frac{2}{3}z^{-2} + \frac{5}{6}z^{-3}$

(d) $y[n] = x[n] - \frac{7}{6}x[n-1] - \frac{2}{3}x[n-2] + \frac{5}{6}x[n-3]$

(e) $H(z)$ can be written as:

$$H(z) = \frac{z^3 - \frac{7}{6}z^2 - \frac{2}{3}z + \frac{5}{6}}{z^3}$$

Zeros at: $z = -\frac{5}{6}, +1, +1$

poles at: $z = 0$ (3 poles)

NOTE: numerator factors into:

$(z-1)(z-1)(z+5/6)$

FACTOR

(f) Need $H_2(z)H_1(z) = 1$

$\therefore\ H_2(z) = \dfrac{1}{H_1(z)} = \dfrac{1}{1 + \frac{5}{6}z^{-1}}$

Need first-order feedback filter to do the DECONVOLUTION