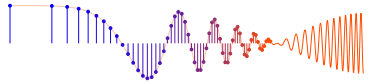

PROBLEM:

Factor the following polynomial and plot its roots in the complex plane.

$$P(z) = 1 + \tfrac{1}{2}z^{-1} + \tfrac{1}{2}z^{-2} + z^{-3}$$

Note the symmetry of the coefficients. In MATLAB see the functions called `roots` and `zplane`.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

$$P(z) = 1 + \tfrac{1}{2}z^{-1} + \tfrac{1}{2}z^{-2} + z^{-3}$$

NOTE $P(-1) = 1 - \tfrac{1}{2} + \tfrac{1}{2} - 1 = 0 \Rightarrow$ ROOT @ $z = -1$

$$\Rightarrow P(z) = \left(1 + z^{-1}\right)\left(1 - \tfrac{1}{2}z^{-1} + z^{-2}\right)$$

USE QUADRATIC FORMULA ON THIS PART

$$\frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4}}{2}$$

$$= \frac{1}{4} \pm j\frac{\sqrt{15}}{4}$$

MAG OF THESE ROOTS IS EXACTLY ONE.