PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = (x[n] + x[n-1] + x[n-2])/3$$

(a) Determine the system function $H(z)$ for this system.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.

(d) What is the output if the input is

$$x[n] = 4 + \cos[0.25\pi(n-1)] - 3\cos[(2\pi/3)n]$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

ANSWERS

(a) $H(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$

(b) Poles at $z = 0$ (2 of them); Zeros at $z = e^{\pm j(2\pi/3)}$

(c) $H(e^{j\hat{\omega}}) = \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) = \frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)} e^{-j\hat{\omega}}$

(d) $y[n] = 4 + 0.8047\cos[0.25\pi(n-2)]$

(a) $H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$

(b) $H(z) = \frac{1}{3}\,\frac{z^2 + z + 1}{z^2}$ ← 2 POLES @ $z=0$

Zeros come from factoring numerator.
USE QUADRATIC FORMULA:

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = -\tfrac{1}{2} \pm j\tfrac{\sqrt{3}}{2} = e^{\pm j 2\pi/3}$$

(c) $H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}} = \frac{1}{3} + \frac{1}{3}e^{-j\hat{\omega}} + \frac{1}{3}e^{-j2\hat{\omega}}$

$$H(e^{j\hat{\omega}}) = \tfrac{1}{3}e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right) = e^{-j\hat{\omega}}\left(\frac{1+2\cos\hat{\omega}}{3}\right)$$

ANOTHER FORMULA WOULD USE "asinc"

$$H(e^{j\hat{\omega}}) = e^{-j\omega}\left(\frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)}\right).$$

(d) Use Linearity & Frequency Response at
$\hat{\omega} = 0,\ \pi/4\ \&\ 2\pi/3.$

$$y[n] = 4\cdot\mathcal{H}(0) + |\mathcal{H}(\pi/4)|\cos\left(\tfrac{\pi}{4}n - \tfrac{\pi}{4} + \angle\mathcal{H}(\tfrac{\pi}{4})\right)$$
$$-3\,\underbrace{|\mathcal{H}(2\pi/3)|}_{=0}\cos\left(\tfrac{2\pi}{3}n + \angle\mathcal{H}(\tfrac{2\pi}{3})\right)$$

$H(e^{j0}) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$H(e^{j\pi/4}) = e^{-j\pi/4}\left(\frac{1 + 2\sqrt{2}/2}{3}\right) = \frac{1+\sqrt{2}}{3}e^{-j\pi/4} = 0.8047\,e^{-j\pi/4}$

$H(e^{j2\pi/3}) = 0$ ← BECAUSE $H(z)$ HAS ZERO AT $e^{\pm j2\pi/3}$

$\therefore\ y[n] = 4 + 0.8047\cos\left(\frac{\pi}{4}n - \frac{2\pi}{4}\right)$