



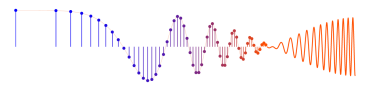
PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = (x[n] + x[n - 1] + x[n - 2])/3$$

- Determine the system function $H(z)$ for this system.
- Plot the poles and zeros of $H(z)$ in the z -plane.
- From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.
- What is the output if the input is

$$x[n] = 4 + \cos[0.25\pi(n - 1)] - 3 \cos[(2\pi/3)n]$$



ANSWERS

(a) $H(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$

(b) Poles at $z = 0$ (2 of them); Zeros at $z = e^{\pm j(2\pi/3)}$

(c) $H(e^{j\hat{\omega}}) = \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) = \frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)}e^{-j\hat{\omega}}$

(d) $y[n] = 4 + 0.8047 \cos[0.25\pi(n - 2)]$

(a) $H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$

(b) $H(z) = \frac{1}{3} \frac{z^2 + z + 1}{z^2}$ 2 POLES @ $z = 0$

Zeros come from factoring numerator.

USE QUADRATIC FORMULA:

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} = e^{\pm j2\pi/3}$$

(c) $H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}} = \frac{1}{3} + \frac{1}{3}e^{-j\hat{\omega}} + \frac{1}{3}e^{-j2\hat{\omega}}$

$$H(e^{j\hat{\omega}}) = \frac{1}{3}e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}) = e^{-j\hat{\omega}} \left(\frac{1 + 2\cos\hat{\omega}}{3} \right)$$

ANOTHER FORMULA WOULD USE "asinc"

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} \left(\frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)} \right)$$

(d) Use Linearity & Frequency Response at $\hat{\omega} = 0, \pi/4$ & $2\pi/3$.

$$y[n] = 4 \cdot \mathcal{H}(0) + \underbrace{|\mathcal{H}(\pi/4)|}_{=0} \cos\left(\frac{\pi}{4}n - \frac{\pi}{4} + \angle\mathcal{H}(\pi/4)\right) - 3 \underbrace{|\mathcal{H}(2\pi/3)|}_{=0} \cos\left(\frac{2\pi}{3}n + \angle\mathcal{H}(2\pi/3)\right)$$

$$H(e^{j0}) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$H(e^{j\pi/4}) = e^{-j\pi/4} \left(\frac{1 + 2\sqrt{2}/2}{3} \right) = \frac{1 + \sqrt{2}}{3} e^{-j\pi/4} = 0.8047 e^{-j\pi/4}$$

$$H(e^{j2\pi/3}) = 0 \leftarrow \text{BECAUSE } H(z) \text{ HAS ZERO AT } e^{\pm j2\pi/3}$$

$$\therefore y[n] = 4 + 0.8047 \cos\left(\frac{\pi}{4}n - \frac{2\pi}{4}\right)$$