PROBLEM:

A linear time-invariant filter is described by the difference equation (with feedback):

$$y[n] = 0.8y[n-1] - 0.8x[n] + x[n-1]$$

(a) Determine the system function $H(z)$ for this system. Express $H(z)$ as a ratio of polynomials in z^{-1} and as a ratio of polynomials in z.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.

(d) Show that $|H(e^{j\hat{\omega}})|^2 = 1$ for all $\hat{\omega}$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $H(z) = \dfrac{-0.8 + z^{-1}}{1 - 0.8z^{-1}}$

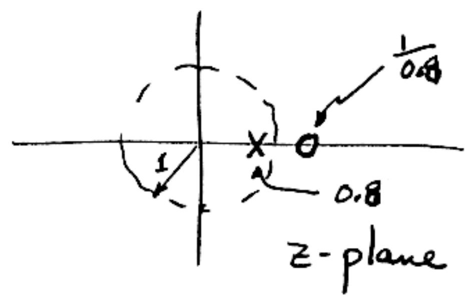
BECAUSE $y[n] - 0.8\,y[n-1] = -0.8\,x[n] + x[n-1]$.

$$Y(z) - 0.8z^{-1}Y(z) = -0.8X(z) + z^{-1}X(z)$$

(b) $H(z) = \dfrac{-0.8(z - 1/0.8)}{z - 0.8}$

ZERO @ $z = 1/0.8 = 1.25$

POLE @ $z = 0.8$



(c) $H(e^{j\hat{\omega}}) = \dfrac{-0.8 + e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$

Sorry, there are no easy simplifications for this one, except as shown in part (d)

(d) $|H(e^{j\hat{\omega}})|^2 = \left(\dfrac{-0.8 + e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}\right)\left(\dfrac{-0.8 + e^{j\hat{\omega}}}{1 - 0.8e^{j\hat{\omega}}}\right) = HH^*$

$$= \frac{-0.8e^{j\hat{\omega}} - 0.8e^{-j\hat{\omega}} + 1.64}{-0.8e^{j\hat{\omega}} - 0.8e^{-j\hat{\omega}} + 1.64} = 1$$

$$= \frac{1.64 - 1.6\cos\hat{\omega}}{1.64 - 1.6\cos\hat{\omega}} = 1$$