



## PROBLEM:

Answer the following questions about the system whose  $z$ -transform system function is

$$H(z) = \frac{1 + 0.8z^{-1}}{1 - 0.9z^{-1}}$$

- (a) Determine the poles and zeros of  $H(z)$ .
- (b) Determine the difference equation relating the input and output of this filter
- (c) Derive a simple expression (purely real) for the mag-squared of the frequency response  $|H(e^{j\hat{\omega}})|^2$ .
- (d) Is this filter a Lowpass or Highpass filter? EXPLAIN your answer.



$$(a) \quad H(z) = \frac{1 + 0.8z^{-1}}{1 - 0.9z^{-1}} = \frac{z + 0.8}{z - 0.9}$$

∴ zero at  $z = -0.8$   
pole at  $z = 0.9$

$$(b) \quad y[n] = 0.9y[n-1] + x[n] + 0.8x[n-1]$$

$$(c) \quad H(e^{j\hat{\omega}}) = \frac{1 + 0.8e^{-j\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}}$$

CONJUGATE

$$|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$$

$$= \frac{1 + 0.8e^{-j\hat{\omega}}}{1 - 0.9e^{-j\hat{\omega}}} \cdot \frac{1 + 0.8e^{+j\hat{\omega}}}{1 - 0.9e^{+j\hat{\omega}}}$$

$$= \frac{1.64 + 0.8e^{+j\hat{\omega}} + 0.8e^{-j\hat{\omega}}}{1.81 - 0.9e^{+j\hat{\omega}} - 0.9e^{-j\hat{\omega}}}$$

$$= \frac{1.64 + 1.6 \cos \hat{\omega}}{1.81 - 1.8 \cos \hat{\omega}} = \begin{cases} 324 & \hat{\omega} = 0 \\ 0.011 & \hat{\omega} = \pi \end{cases}$$

(d) It is a  
LOWPASS  
Filter  
see plot →

