PROBLEM:

Given a feedback filter defined via the recursion:

$$y[n] = \tfrac{2}{3}y[n-1] + 6x[n] \qquad \text{(DIFFERENCE EQUATION)}$$

(a) Determine the system function $H(z)$. What are its poles and zeros?

(b) When the input to the system is a short alternating signal:

$$x[n] = \begin{cases} (-1)^n & \text{when } n = 0, 1, 2, 3 \\ 0 & \text{when } n < 0 \text{ and } n \geq 4 \end{cases}$$

determine the functional form for the output signal $y[n]$. Assume that the output signal $y[n]$ is zero for $n < 0$. This is called the *at rest* initial condition for the difference equation.
(Optional) Use the `filter( )` function in MATLAB to check your answer.

(c) Write a formula for the input signal $x[n]$ as the weighted sum of four shifted impulse signals.

(d) Explain how it is possible to use linearity to express the output as the sum of four terms, each of which is a delayed version of the impulse response of the system. Recall that the impulse response of a first-order feedback filter has the generic form: $b_0(a_1)^n$ for $n \geq 0$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $y[n] = \frac{2}{3}y[n-1] + 6x[n]$ ← IN MATLAB: $b=[6]$ $a=[1\ \ -2/3]$

$$H(z) = \frac{6}{1-\frac{2}{3}z^{-1}} = \frac{6z}{z-\frac{2}{3}}$$

∴ ZERO @ $z=0$; POLE @ $z=2/3$

(b) MAKE A TABLE:

n	$x[n]$	$y[n]$	
<0	0	0	
0	1	6	$y[0] = \frac{2}{3}(0) + 6(1) = 6$
1	-1	-2	$y[1] = \frac{2}{3}(6) + 6(-1) = -2$
2	1	14/3	$y[2] = \frac{2}{3}(-2) + 6(1) = 14/3$
3	-1	-26/9	$y[3] = \frac{2}{3}(14/3) + 6(-1) = -26/9$
4	0	-52/27	$y[4] = \frac{2}{3}(-26/9) + 6(0) = -52/27$
5	0	-104/81	$y[5] = \frac{2}{3}(-52/27) + 6(0) = -104/81$
⋮	⋮	⋮	

$$y[n] = \begin{cases} 0 & \text{for } n<0 \\ 6 & \text{for } n=0 \\ -2 & \text{for } n=1 \\ 14/3 & \text{for } n=2 \\ -\frac{26}{9}\left(\frac{2}{3}\right)^{n-3} & \text{for } n\geq 3 \end{cases}$$

$y[n]$ behaves like $\left(\frac{2}{3}\right)^{n-3}$ for $n\geq 3$

NOTE: pole is at $z=\frac{2}{3}$

(d) Find the impulse response $h[n]$.

$\delta[n] \rightarrow \boxed{S} \rightarrow h[n]$ $\qquad h[n] = 6\left(\frac{2}{3}\right)^n$ for $n\geq 0$

Since $x[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$

we get $y[n] = \underbrace{h[n] - h[n-1] + h[n-2] - h[n-3]}_{\text{SUM OF FOUR TERMS}}$

NOTE: $h[n-2] = 6\left(\frac{2}{3}\right)^{n-2}$ for $n\geq 2$

MUST SHIFT THE STARTING POINT

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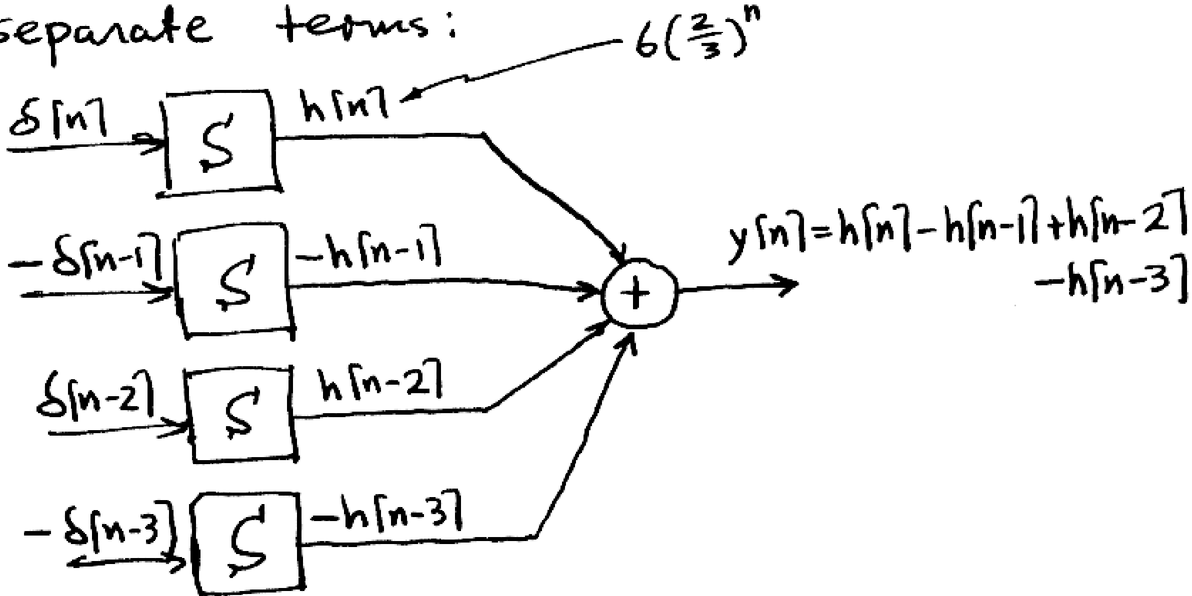
Prob (cont)

(d) Further explanation of "SUPERPOSITION"

Since $x[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$

we can construct the output as four

separate terms:



To get a formula, you have to keep track of the starting points

For $n \geq 3$:

$$y[n] = 6\left(\tfrac{2}{3}\right)^n - 6\left(\tfrac{2}{3}\right)^{n-1} + 6\left(\tfrac{2}{3}\right)^{n-2} - 6\left(\tfrac{2}{3}\right)^{n-3}$$

$$= \left[6 - 6\left(\tfrac{2}{3}\right)^{-1} + 6\left(\tfrac{3}{2}\right)^2 - 6\left(\tfrac{3}{2}\right)^3\right]\left(\tfrac{2}{3}\right)^n = \frac{-39}{4}\left(\tfrac{2}{3}\right)^n$$

For $n = 2$ (use 3 terms)

$$y[2] = 6\left(\tfrac{2}{3}\right)^2 - 6\left(\tfrac{2}{3}\right)^1 + 6\left(\tfrac{2}{3}\right)^0 = 14/3$$

For $n = 1$ (use 2 terms)

$$y[1] = 6\left(\tfrac{2}{3}\right) - 6\left(\tfrac{2}{3}\right)^0 = -2$$

For $n = 0$ (use one term)

$$y[0] = 6\left(\tfrac{2}{3}\right)^0 = 6$$