PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = 0.8y[n-1] - 0.8x[n] + x[n-1]$$

(a) Determine the system function $H(z)$ for this system. Express $H(z)$ as a ratio of polynomials in z^{-1} (negative powers of z) and also as a ratio of polynomials in positive powers of z.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.

(d) Show that $|H(e^{j\hat{\omega}})|^2 = 1$ for all $\hat{\omega}$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

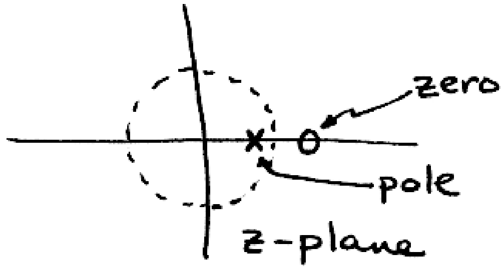
SOLUTION

(a) $H(z) = \dfrac{-0.8 + z^{-1}}{1 - 0.8z^{-1}}$ BY PICKING THE COEFFS FROM THE DIFF. EQN.

(b) POLE @ $z = 0.8$

ZERO @ $z = 1/0.8$

$= 1.25$



(c) $H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$

$$= \frac{-0.8 + e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$$

(d) $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})\, H^*(e^{j\hat{\omega}})$ MULTIPLY BY CONJUGATE

$$= \frac{(-0.8 + e^{-j\hat{\omega}})(-0.8 + e^{+j\hat{\omega}})}{(1 - 0.8e^{-j\hat{\omega}})(1 - 0.8e^{+j\hat{\omega}})}$$

$$= \frac{.64 + 1 - 0.8e^{-j\hat{\omega}} - 0.8e^{+j\hat{\omega}}}{1 + .64 - 0.8e^{-j\hat{\omega}} - 0.8e^{+j\hat{\omega}}}$$

$$= \frac{1.64 - 1.6\cos\hat{\omega}}{1.64 - 1.6\cos\hat{\omega}}$$

$\therefore\ |H(e^{j\hat{\omega}})|^2 = 1$ for all $\hat{\omega}$