PROBLEM:

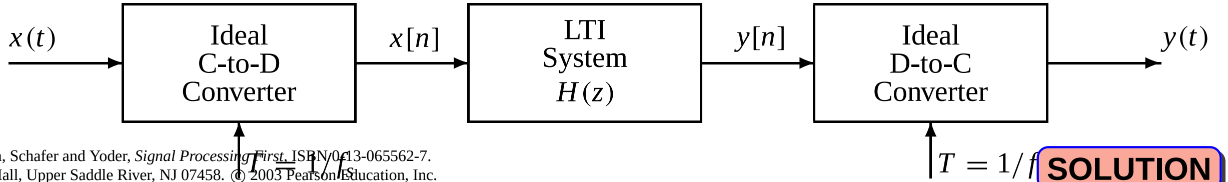
The input to the C-to-D converter in the figure below is

$$x(t) = 7 + 9\cos(1600\pi t - \pi/4) - 11\cos(12000\pi t - \pi/3)$$

The system function for the LTI system is

$$H(z) = \frac{1}{5}(1 + z^{-5})$$

If $f_s = 8000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



SOLUTION

$$x[n] = x(t)\Big|_{t=n/f_s} = x(t)\Big|_{t=n/8000}$$

$$= 7 + 9\cos\left(1600\pi\frac{n}{8000} - \pi/4\right) - 11\cos\left(12000\pi\frac{n}{8000} - \pi/3\right)$$

$$= 7 + 9\cos\left(\frac{\pi}{5}n - \pi/4\right) - 11\cos\left(\frac{3\pi}{2}n - \pi/3\right)$$

ALIAS TO $\pi/2$
ACTUALLY
FOLDING

$$x[n] = 7 + 9\cos\left(2\pi\left(\tfrac{1}{10}\right)n - \pi/4\right) - 11\cos\left(2\pi\left(\tfrac{1}{4}\right)n + \frac{\pi}{3}\right)$$

Need to evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = 0$, $2\pi\left(\frac{1}{10}\right)$, & $2\pi\left(\frac{1}{4}\right)$

$$H(e^{j\hat{\omega}}) = \frac{1}{5}\left(1 + e^{-j5\hat{\omega}}\right)$$

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=0} = \frac{1}{5}\left(1 + e^{j0}\right) = \frac{2}{5}$$ NO PHASE

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\pi/5} = \frac{1}{5}\left(1 + e^{-j5(\pi/5)}\right) = \frac{1}{5}\left(1 + e^{-j\pi}\right) = 0$$

$$H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\pi/2} = \frac{1}{5}\left(1 + e^{-j5\pi/2}\right) = \frac{1}{5}(1 - j) = \frac{\sqrt{2}}{5}e^{-j\pi/4}$$

$$\therefore\ y[n] = \frac{2}{5}(7) + \underbrace{9(0)\cos\left(\frac{\pi}{5}n - \pi/4\right)}_{\text{ZERO}} - 11\left(\frac{\sqrt{2}}{5}\right)\cos\left(\frac{\pi}{2}n + \frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= 2.8 + 3.11\cos\left(2\pi\left(\tfrac{1}{4}\right)n + \frac{\pi}{12}\right)$$

Convert back to continuous time by replacing n with $f_s t$, i.e., $n \leftarrow 8000t$

$$\therefore\ y(t) = 2.8 + 3.11\cos\left(2\pi(2000)t + \pi/12\right)$$

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.