PROBLEM:

Define a discrete-time signal via the formula:

$$y[n] = 7(0.9)^n \cos(2\pi(0.2)n + \phi)$$

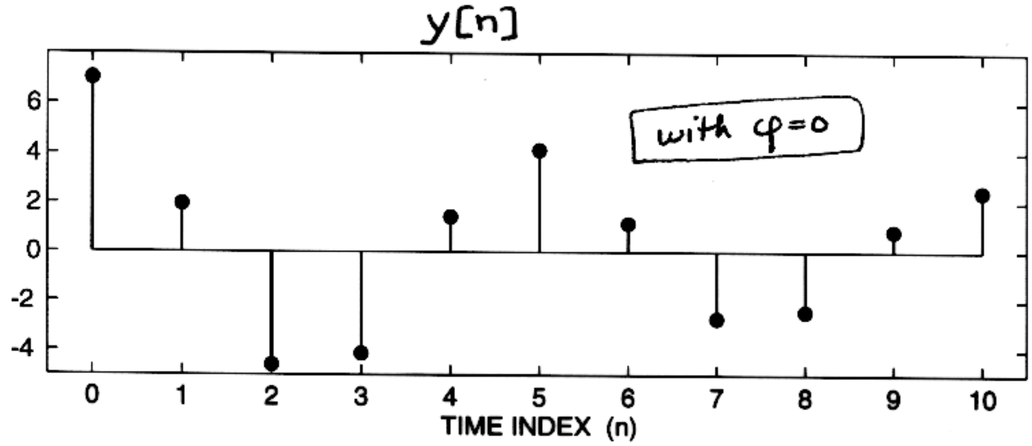
(a) Make a sketch of $y[n]$ versus n, as a "comb" plot. Take the range of n to be $0 \le n \le 10$.

(b) Design a (second-order) feedback filter that will synthesize $y[n]$. Give your answer in the form of a difference equation with numerical values for the coefficients. Assume that the synthesis will be accomplished by using an impulse input to "start" the difference equation (which is at rest, i.e., has zero initial conditions).

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a)



(b) Need poles at $z = 0.9e^{j2\pi(0.2)}$ & $z = 0.9e^{-j2\pi(0.2)}$

$$\Rightarrow A(z) = \left(1 - 0.9e^{j0.4\pi}z^{-1}\right)\left(1 - 0.9e^{-j0.4\pi}z^{-1}\right)$$

$$= 1 - \left(0.9e^{j0.4\pi} + 0.9e^{-j0.4\pi}\right)z^{-1} + (0.9)^2 z^{-2}$$

$$= 1 - \underbrace{1.8\cos(0.4\pi)}_{0.556} z^{-1} + 0.81 z^{-2}$$

$\Rightarrow$ difference equation:

$$y[n] = 0.556\,y[n-1] - 0.81\,y[n-2] + G\,x[n]$$

NEED TO ADJUST GAIN G to get the 7 in y[n].

If we calculate the impulse response, we get

n	0	1	2	3	4	5	...	8	9	10	...
$h[n]$	G	$.556G$	$-.5G$	$-.73G$	0	$.59G$	...	$-.43G$	0	$.35G$	...

From the zero crossings we find a formula for $h[n]$

$$h[n] = A\,(0.9)^n \cos\left(\tfrac{2\pi}{5}(n-4) - \tfrac{\pi}{2}\right) = A\,(0.9)^n \cos\left(\tfrac{2\pi}{5}n \underbrace{-\,0.1\pi}_{\varphi = -\frac{\pi}{10}}\right)$$

To make the $n=0$ point correct:

$$h[0] = G = A\cos(-0.1\pi) = 7\cos(0.1\pi) \Rightarrow G = 6.66$$