



PROBLEM:

Suppose that three systems are hooked together in “cascade.” In other words, the output of \mathcal{S}_1 is the input to \mathcal{S}_2 , and the output of \mathcal{S}_2 is the input to \mathcal{S}_3 . The three systems are specified as follows:

$$\mathcal{S}_1 : \quad y_1[n] = x_1[n] - x_1[n - 1]$$

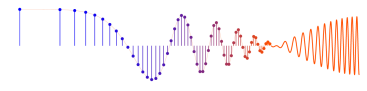
$$\mathcal{S}_2 : \quad y_2[n] = x_2[n] + x_2[n - 2]$$

$$\mathcal{S}_3 : \quad y_3[n] = x_3[n - 1] + x_3[n - 2]$$

NOTE: the output of \mathcal{S}_i is $y_i[n]$ and the input is $x_i[n]$.

Determine the equivalent system that is a single operation from the input $x[n]$ (into \mathcal{S}_1) to the output $y[n]$ which is the output of \mathcal{S}_3 . Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

Write one difference equation that defines the overall system in terms of $x[n]$ and $y[n]$ only..



$$\begin{aligned} S_1: H_1(z) &= 1 - z^{-1} \\ S_2: H_2(z) &= 1 + z^{-2} \\ S_3: H_3(z) &= z^{-1} + z^{-2} \end{aligned}$$

NOW, MULTIPLY
ALL THREE
TOGETHER.

$$\begin{aligned} H(z) &= (1 - z^{-1})(1 + z^{-2})(z^{-1} + z^{-2}) \\ &= (1 - z^{-1} + z^{-2} - z^{-3})(1 + z^{-1})z^{-1} \\ &= z^{-1}(1 - z^{-1} + z^{-2} - z^{-3} + z^{-1} - z^{-2} + z^{-3} - z^{-4}) \\ &= z^{-1}(1 - z^{-4}) \\ &= z^{-1} - z^{-5} \end{aligned}$$

$$\therefore y[n] = x[n-1] - x[n-5].$$

THIS IS
 $y_3[n]$

INPUT WAS
 $x_1[n]$