PROBLEM:

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\mathcal{S}_1 : \qquad y_1[n] = x_1[n] - x_1[n-1]$$

$$\mathcal{S}_2 : \qquad y_2[n] = x_2[n] + x_2[n-2]$$

$$\mathcal{S}_3 : \qquad y_3[n] = x_3[n-1] + x[n-2]$$

NOTE: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

Determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

Write one difference equation that defines the overall system in terms of $x[n]$ and $y[n]$ only..

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

$S_1: \quad H_1(z) = 1 - z^{-1}$

$S_2: \quad H_2(z) = 1 + z^{-2}$

$S_3: \quad H_3(z) = z^{-1} + z^{-2}$

NOW, MULTIPLY ALL THREE TOGETHER.

$$H(z) = (1 - z^{-1})(1 + z^{-2})(z^{-1} + z^{-2})$$

$$= (1 - z^{-1} + z^{-2} - z^{-3})(1 + z^{-1})\, z^{-1}$$

$$= z^{-1}(1 - z^{-1} + z^{-2} - z^{-3} + z^{-1} - z^{-2} + z^{-3} - z^{-4})$$

$$= z^{-1}(1 - z^{-4})$$

$$= z^{-1} - z^{-5}$$

$\therefore$

$$y[n] = x[n-1] - x[n-5].$$

THIS IS $y_3[n]$

INPUT WAS $x_1[n]$

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