PROBLEM:

Suppose that a system is defined by the following operator

$$H(z) = (1 - z^{-1})\frac{1 + z^{-4}}{1 - 0.8z^{-2}}$$

(a) Write the time-domain description of this system—in the form of a difference equation.

(b) Write the formula for the frequency response of the system.

(c) Derive a simple formula for the magnitude squared of $H(e^{j\hat{\omega}})$ response versus $\hat{\omega}$.

(d) This system can "block" certain input signals. For which input frequencies $\omega_\circ$, is the response to $x[n] = \cos(\omega_\circ n)$ equal to zero?

(e) When the input to the system is $x[n] = \cos(\pi n/3)$ determine the output signal $y[n]$ in the form:

$$A\cos(\omega_\circ n + \phi)$$

Give numerical values for the constants A, $\omega_\circ$ and ϕ.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a) $H(z) = \dfrac{1 - z^{-1} + z^{-4} - z^{-5}}{1 - 0.8z^{-2}}$ ← use coeffs. to make DIFFERENCE EQN

$y[n] = 0.8y[n-2] + x[n] - x[n-1] + x[n-4] - x[n-5]$

(b) $H(e^{j\hat{\omega}}) = (1 - e^{-j\hat{\omega}})\dfrac{1 + e^{-j4\hat{\omega}}}{1 - 0.8e^{-j2\hat{\omega}}}$ ← BY REPLACING z with $e^{j\hat{\omega}}$

(c) $|H(e^{j\hat{\omega}})|^2 = \dfrac{|1 - e^{-j\hat{\omega}}|^2 \, |1 + e^{-j4\hat{\omega}}|^2}{|1 - 0.8e^{-j2\hat{\omega}}|^2}$ ← simplify each term

NOTE: $|1 + ae^{-jN\hat{\omega}}|^2 = (1 + ae^{-jN\hat{\omega}})(1 + ae^{+jN\hat{\omega}})$

$= 1 + a^2 + ae^{-jN\hat{\omega}} + ae^{jN\hat{\omega}} = 1 + a^2 + 2a\cos N\hat{\omega}$

$\therefore \; |H(e^{j\hat{\omega}})|^2 = \dfrac{(2 - 2\cos\hat{\omega})(2 + 2\cos 4\hat{\omega})}{1.64 - 1.6\cos 2\hat{\omega}}$

(d) Find the zeros and see if they are on the unit circle:

$(1 - z^{-1})(1 + z^{-4}) = 0 \Rightarrow$ $1 - z^{-1} = 0 \Rightarrow z = 1$

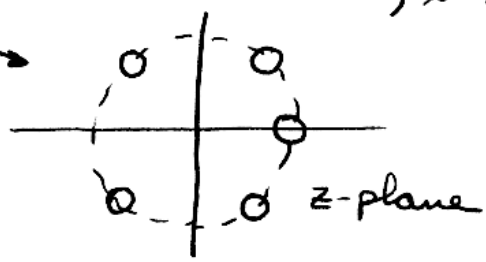
$1 + z^{-4} = 0 \Rightarrow z^4 = -1 = e^{j\pi}e^{j2\pi\ell}$

$\Rightarrow z = e^{j(\pi/4 + \pi\ell/2)}, \; \ell = 0, 1, 2, 3$

ALL FIVE ZEROS ARE ON THE UNIT CIRCLE

$\therefore \cos\hat{\omega}_0 n$ is BLOCKED when $\hat{\omega}_0 = 0, \pi/4$ & $+3\pi/4$

(NEG. FREQS ARE THE SAME)



(e) when $x[n] = \cos(\frac{\pi}{3}n)$ the output is determined by $|H(e^{j\hat{\omega}})|$ and $\angle H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$.

$\mathcal{H}(\pi/3) = \dfrac{(1 - e^{-j\pi/3})(1 + e^{-j4\pi/3})}{1 - 0.8e^{-j2\pi/3}} = \dfrac{(1e^{j\pi/3})(1e^{j\pi/3})}{1.56e^{j0.146\pi}} = 0.64e^{j0.52\pi}$

93.67°

$\Rightarrow y[n] = 0.64\cos(\frac{\pi}{3}n + 0.52\pi)$