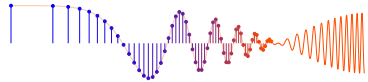


PROBLEM:

For each of the following systems (specified by either an $H(z)$ or a difference equation), determine all the poles and zeros and make a pole-zero plot.

(a) $\mathcal{S}_a$: $\quad y[n] = 5x[n] - 10x[n-2] + 5x[n-4]$

(b) $\mathcal{S}_b$: $\quad H(z) = \dfrac{3 + 3z^{-1} + 3^{-2}}{1 + 0.6z^{-1} + 0.81z^{-2}}$

(c) $\mathcal{S}_c$: $\quad y[n] = -0.81y[n-2] + 4x[n-1] - 5x[n-2]$

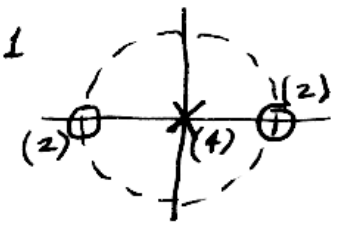
McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $H(z) = 5 - 10z^{-2} + 5z^{-4} = \dfrac{5(z^4 - 2z^2 + 1)}{z^4} = \dfrac{5(z^2-1)^2}{z^4}$

$(z^2-1)^2 = 0$ HAS FOUR ZEROS:
TWO AT $z = 1$, TWO AT $z = -1$

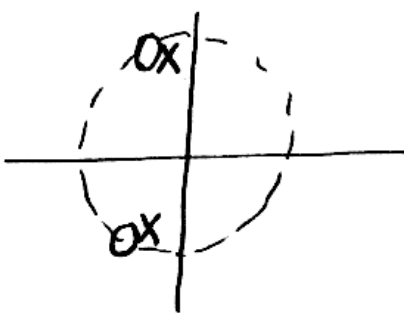
THERE ARE FOUR POLES at $z = 0$



(b) $H(z) = \dfrac{3(z^2+z+1)}{z^2+0.6z+0.81}$

ROOTS of $z^2+z+1=0$ ← ZEROS

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} = 1e^{\pm j 2\pi/3}$$



ROOTS of $z^2+0.6z+0.81=0$

$$z = \frac{-0.6 \pm \sqrt{.36-3.24}}{2} = -0.3 \pm j0.6\sqrt{2} = -0.3 \pm j0.4242$$

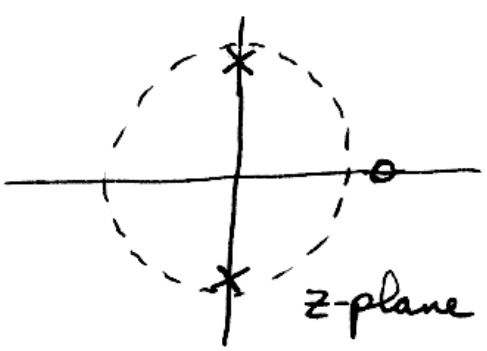
(c) $H(z) = \dfrac{4z^{-1} - 5z^{-2}}{1 + 0.81z^{-2}} = \dfrac{4z-5}{z^2+0.81}$

ROOTS of $z^2 + 0.81 = 0$

$z = \pm j0.9$ ← POLES

ROOTS of $4z - 5 = 0$

$z = 5/4$ ← ZERO



ALSO

$$\lim_{z\to\infty} \frac{4z-5}{z^2+0.81} \to \frac{4z}{z^2} \to \frac{4}{z} \to 0$$