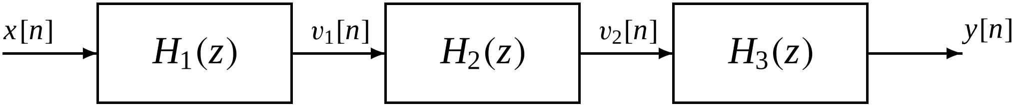
PROBLEM:

In the following cascade of systems, all of the individual transfer functions are known.



$$H_1(z) = z^{-2} + z^{-3} \qquad H_2(z) = 4 - 3z^{-1} \qquad H_3(z) = \frac{10}{8 - 10z^{-1} + 3z^{-2}}$$

(a) Find the second output $v_2[n]$ when the input signal $x[n]$ is an impulse, i.e., $x[n] = \delta[n]$. Give a general formula in terms of α and β for $n \geq 0$.

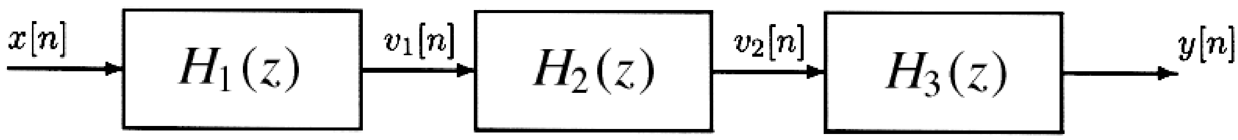
(b) Determine $H(z)$ the z-transform of the cascaded system. Simplify $H(z)$ by factoring the numerator and denominator.

(c) Consider the impulse response of the cascaded system, i.e., the response $y[n]$ when the input is $x[n] = \delta[n]$. Prove that the impulse response has the form $h[n] = G\,\alpha^n$ for $n \geq 4$. Find values for α and G.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

In the following cascade of systems, all of the individual transfer functions are known.



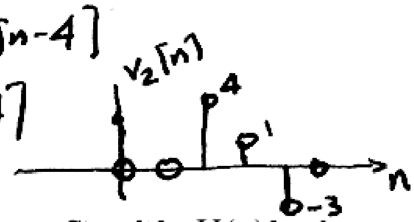
$$H_1(z) = z^{-2} + z^{-3} \qquad H_2(z) = 4 - 3z^{-1} \qquad H_3(z) = \frac{10}{8 - 10z^{-1} + 3z^{-2}}$$

(a) Find the second output $v_2[n]$ when the input signal $x[n]$ is an impulse, i.e., $x[n] = \delta[n]$.

$$H_1(z)H_2(z) = (z^{-2} + z^{-3})(4 - 3z^{-1})$$
$$= 4z^{-2} + z^{-3} - 3z^{-4}$$

$\Rightarrow$ when $x[n] = \delta[n]$

$$v_2[n] = 4x[n-2] + x[n-3] - 3x[n-4]$$
$$= 4\delta[n-2] + \delta[n-3] - 3\delta[n-4]$$



(b) Determine $H(z)$ the z-transform of the cascaded system. Simplify $H(z)$ by factoring the numerator and denominator.

$$H(z) = H_1(z)H_2(z)H_3(z)$$ ← MULTIPLY THE TRANSFER FCNS

$$H(z) = \frac{(z^{-2} + z^{-3})(4 - 3z^{-1})\,10}{8 - 10z^{-1} + 3z^{-2}}$$ ← FACTOR DENOM: $(4 - 3z^{-1})(2 - z^{-1})$

$$H(z) = \frac{10(z^{-2} + z^{-3})}{2 - z^{-1}}$$ ← CANCEL THE TERM $(4 - 3z^{-1})$ FROM BOTH NUM & DENOM

(c) Consider the impulse response of the cascaded system, i.e., the response $y[n]$ when the input is $x[n] = \delta[n]$. Prove that the impulse response has the form $h[n] = G\alpha^n$ for $n \geq 4$. Find values for α and G.

USE $\mathcal{H}(z)$ TO GET DIFFERENCE EQN FOR OVERALL SYSTEM:

$$y[n] = \tfrac{1}{2}y[n-1] + 5x[n-2] + 5x[n-3].$$

$G = 60$ | $n \geq 3$

$y[n] = 60\left(\tfrac{1}{2}\right)^n$

POLE @ $z = 1/2$

$\Rightarrow \left(\tfrac{1}{2}\right)^n \quad \therefore \; \alpha = \tfrac{1}{2}$

SOLVE DIFF. EQN: $x[n] = \delta[n]$.

$y[2] = 5 \qquad y[4] = 3.75$

$y[3] = 7.5 \qquad y[5] = \tfrac{1}{2}y[4]$.

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