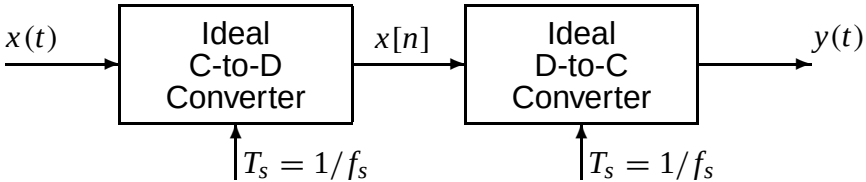
PROBLEM:

Consider the following system.

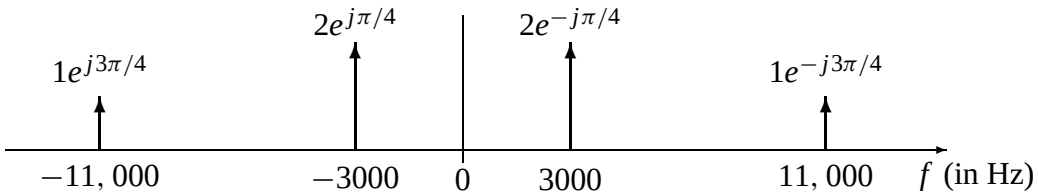


(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 27\cos(0.11\pi n - \pi/3)$$

If the sampling rate is $f_s = 5000$ samples/second, determine two *different* continuous-time signals $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have been inputs to the above system; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 200$ μsec. Both of these input signals should have a frequency less than 5000 Hz. Give a formula for each signal.

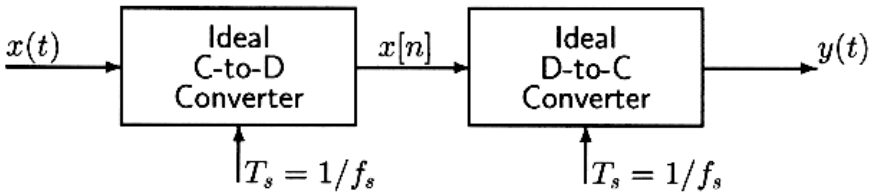
(b) If the input $x(t)$ is given by the two-sided spectrum representation shown below, determine a simple formula for $y(t)$ when $f_s = 5000$ samples/sec. (for both the C/D and D/C converters).



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Consider the following system.



(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 27\cos(0.11\pi n - \pi/3) = 27\cos\left(2\pi(0.055)n - \tfrac{\pi}{3}\right)$$

If the sampling rate is $f_s = 5000$ samples/second, determine two *different* continuous-time signals $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have been inputs to the above system; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 200$ μsec. Both of these input signals should have a frequency less than 5000 Hz. Give a formula for each signal.

Let $x_1(t) = A_1\cos(2\pi F_1 t + \varphi_1)$ $\quad A_1 = 27 \quad \varphi_1 = -\pi/3$

$\Rightarrow x_1[n] = A_1\cos\left(2\pi F_1 \frac{n}{5000} + \varphi_1\right) \Rightarrow 2\pi\frac{F_1}{5000} = 2\pi(0.055) \Rightarrow F_1 = 275\,\text{Hz}$

The other case is folding:

$2\pi\left(\frac{F_2}{5000}\right) = 2\pi(1-0.055)$

$\Rightarrow F_2 = 5000 - 275$

$= 4725\,\text{Hz}.$

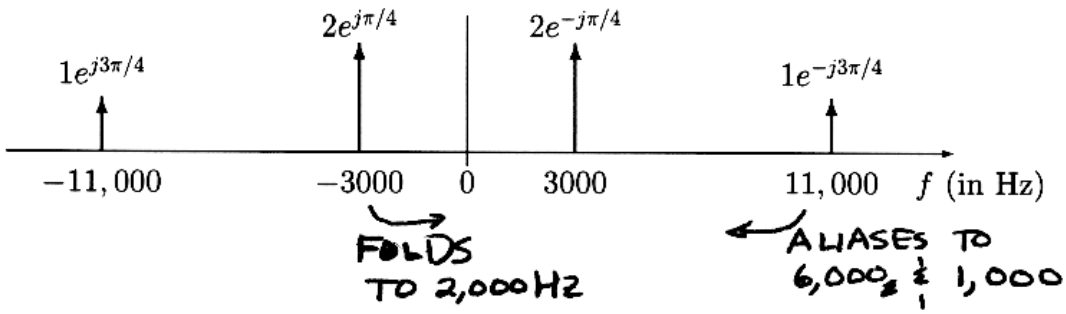
$A_2\cos(2\pi(4725)t + \varphi_2) = x_2(t)$

$A_2 = 27$

Need sign change on phase

$\varphi_2 = +\pi/3$

(b) If the input $x(t)$ is given by the two-sided spectrum representation shown below, determine a simple formula for $y(t)$ when $f_s = 5000$ samples/sec. (for both the C/D and D/C converters).



$x(t) = 2\cos(2\pi(11{,}000)t - 3\pi/4) + 4\cos(2\pi(3000)t - \pi/4)$

$x[n] = 2\cos\left(2\pi\left(\frac{11000}{5000}\right)n - 3\pi/4\right) + 4\cos\left(2\pi\left(\frac{3000}{5000}\right)n - \pi/4\right)$

$x[n] = 2\cos(2\pi(2.2)n - 3\pi/4) + 4\cos(2\pi(0.6)n - \pi/4)$

$x[n] = 2\cos(2\pi(0.2)n - 3\pi/4) + 4\cos(2\pi(-0.4)n - \pi/4)$

$4\cos(2\pi(0.4)n + \pi/4)$

use $n \to 5000t$ for D/C

$y(t) = 2\cos(2\pi(1000)t - 3\pi/4) + 4\cos(2\pi(2000)t + \pi/4)$