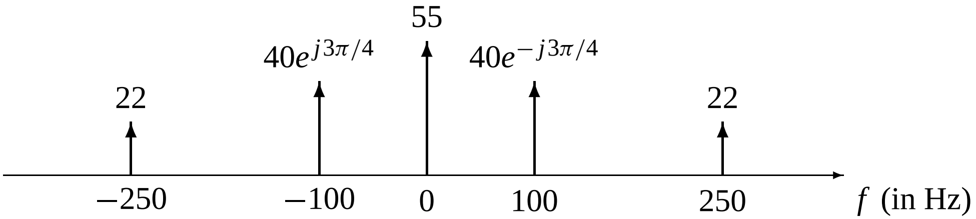
PROBLEM:

A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x(t)$. Make sure to express $x(t)$ as a real-valued signal.

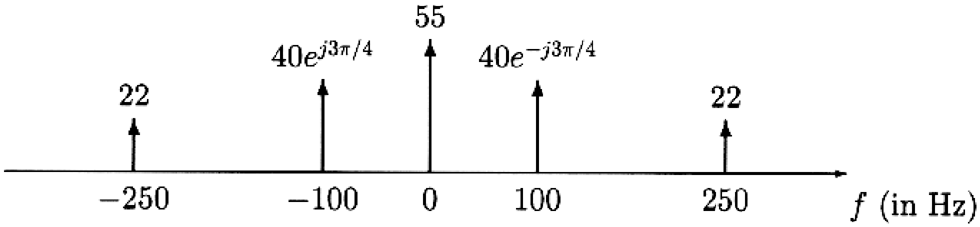
(b) The signal $x(t)$ is periodic—determine its (minimum) period.

(c) Determine the minimum sampling rate that can be used to sample $x(t)$ without any aliasing.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

A signal $x(t)$ has the two-sided spectrum representation shown below.



(a) Write an equation for $x(t)$. Make sure to express $x(t)$ as a real-valued signal.

$$x(t) = 22e^{-j2\pi(250)t} + 40e^{j3\pi/4}e^{-j2\pi(100)t} + 55$$
$$+22e^{+j2\pi(250)t} + 40e^{-j3\pi/4}e^{+j2\pi(100)t}$$

COMBINE POSITIVE & NEGATIVE FREQ PARTS:

$$x(t) = 44\cos(2\pi(250)t) + 80\cos\left(2\pi(100)t - \tfrac{3\pi}{4}\right) + 55$$

(b) The signal $x(t)$ is periodic—determine its (minimum) period.

Method 1: Find Fundamental Freq F_0 which divides both 100 Hz & 250 Hz. ANS: 50 Hz

$\Rightarrow$ $\boxed{\text{period} = \frac{1}{50}\text{ sec}}$

Method 2: Find common period.

$$\frac{1}{100}\ell_1 = \frac{1}{250}\ell_2$$

$$\frac{250}{100} = \frac{\ell_2}{\ell_1} = 2.5 = \frac{5}{2}$$

$\therefore\ \ell_2 = 5$ & $\ell_1 = 2$

so period $= \frac{2}{100}$ or $\frac{5}{250} = \frac{1}{50}$ sec.

NOTE: the constant 55 does NOT affect the period.

(c) Determine the minimum sampling rate that can be used to sample $x(t)$ without any aliasing.

MIN SAMPLING RATE > 2 F_{HIGHEST}.

$$\therefore\ F_{\text{SAMP}} > 2(250\text{ Hz}) = 500\text{ samples/sec.}$$

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