PROBLEM:

The diagram in Figure 1 depicts a *cascade connection* of two linear time-invariant systems.

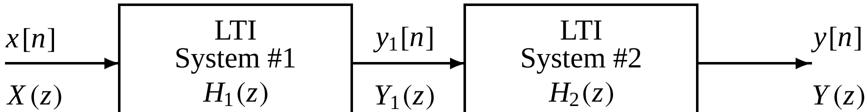
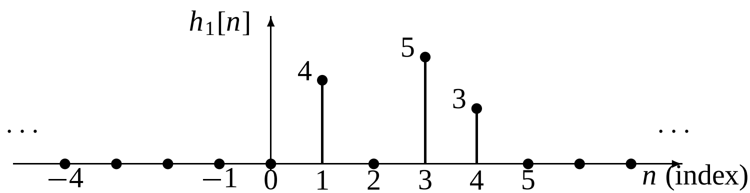


Figure 1: Cascade connection of two LTI systems.

(a) Determine the filter coefficients $\{b_k\}$ of the first system: $y_1[n] = \sum_{k=0}^{M} b_k\, x[n-k]$

Assume that the impulse response from the first filter is the signal $h_1[n]$ shown below:

$$\delta[n] = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases} \quad \longrightarrow$$



State clearly the value for the order M as well as all the coefficients.

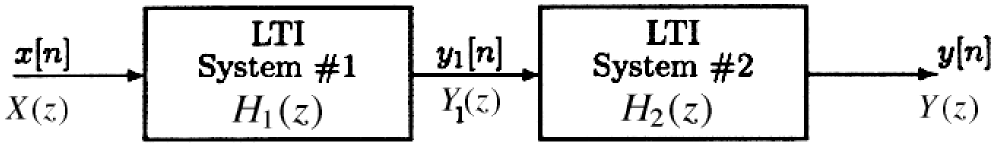
(b) Suppose that system #2 is described by the system function: $H_2(z) = \dfrac{10}{1 - \frac{2}{5}z^{-1}}$

Obtain the impulse response of the cascade, i.e., find $y[n]$ when $x[n] = \delta[n]$ in Figure 1.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

The diagram in Figure 1 depicts a *cascade connection* of two linear time-invariant systems.



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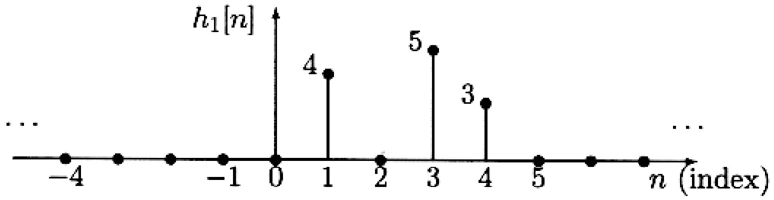

State clearly the value for the order M as well as all the coefficients.

$M=4$, so length of FIR filter is 5

Since impulse response "reads out" the $\{b_k\}$ we get

$$\{b_k\} = \{0, 4, 0, 5, 3\}$$

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Obtain the impulse response of the cascade, i.e., find $y[n]$ when $x[n] = \delta[n]$ in Figure 1.

$H_2(z) \Rightarrow \quad y[n] = \frac{2}{5}y[n-1] + 10\,y_1[n]$ is SYSTEM #2

We already have impulse response of SYS #1, so use $h_1[n]$ as input to SYS #2. MAKE TABLE

n	<0	0	1	2	3	4	5	6	
$h_1[n]$	0	0	4	0	5	3	0	0	
$y[n]$	0	0	40	16	56.4	52.56	21.024	....	

for $n \geq 4$ $\quad y[n] = 52.56\left(\frac{2}{5}\right)^{n-4}$

for $n=3$ $\quad y[n] = 56.4$

$n=2$ $\quad y[n] = 16$

$n=1$ $\quad y[n] = 40$

$n \leq 0$ $\quad y[n] = 0$

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