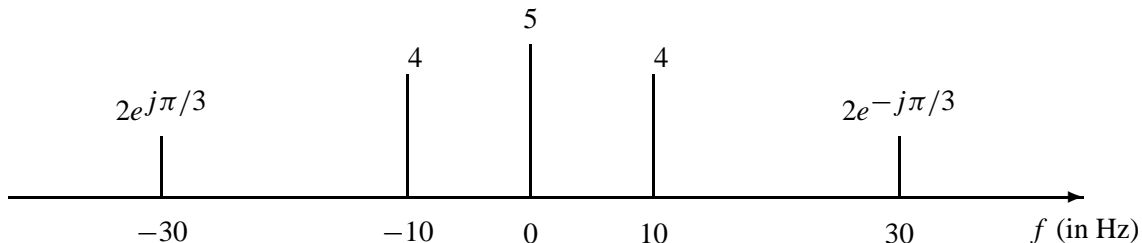


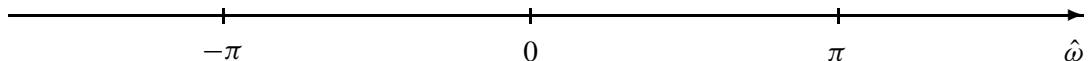


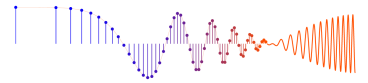
PROBLEM:

A signal $x(t)$ has the two-sided spectrum representation shown below.



- Write an equation for $x(t)$.
- Is the signal $x(t)$ periodic? If so, what is the period?
- The signal $x(t)$ is sampled with sampling frequency $f_s = 1/T_s = 50$ samples/second to obtain the discrete-time signal $x[n] = x(nT_s)$. Write an equation for $x[n]$ and plot the spectrum of $x[n]$ for normalized frequencies $-\pi \leq \hat{\omega} \leq \pi$.





$$(a) x(t) = 2e^{j\pi/3} e^{-j2\pi(30)t} + 4e^{-j2\pi(10)t} + 4e^{j2\pi(10)t} + 5$$

$$+ 2e^{-j\pi/3} e^{j2\pi(30)t} \quad \underbrace{\hspace{10em}}_{8\cos(2\pi(10)t)}$$

$$x(t) = 4\cos(60\pi t - \pi/3) + 8\cos(20\pi t) + 5$$

(b) $\cos(60\pi t) \rightarrow \text{period} = 2\pi/60\pi = 1/30 \text{ sec}$

$\cos(20\pi t) \rightarrow \text{period} = 2\pi/20\pi = 1/10 \text{ sec.}$

Since $1/30$ divides $1/10$, the period of $x(t)$ is $1/10 \text{ sec}$

OR 30 Hz is three times 10 Hz, so the fundamental frequency is 10 Hz \Rightarrow period = $1/10 \text{ sec.}$

(c) $F_s = 1/T_s = 50$

$$x[n] = x(nT_s) = x(n/F_s)$$

$$= 4\cos(60\pi n/50 - \pi/3) + 8\cos(20\pi n/50) + 5$$

$$= 4\cos\left(\frac{6\pi n}{5} - \pi/3\right) + 8\cos\left(\frac{2\pi}{5}n\right) + 5 \quad \left(\frac{6\pi}{5} = 2\pi - \frac{4\pi}{5}\right)$$

$$= 4\cos\left(2\pi n - \frac{4\pi}{5}n - \pi/3\right) + 8\cos\left(\frac{2\pi}{5}n\right) + 5$$

$$= 4\cos\left(\frac{4\pi}{5}n + \pi/3\right) + 8\cos\left(\frac{2\pi}{5}n\right) + 5$$

