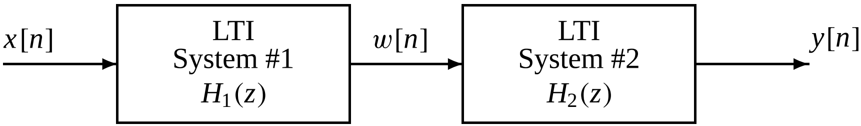
PROBLEM:



The overall system function of the above system (from input $x[n]$ to output $y[n]$) is

$$H(z) = \frac{(1 - z^{-1})(1 + z^{-2})}{1 + 0.8z^{-1}}$$

(a) Determine system functions $H_1(z)$ and $H_2(z)$ such that System #1 is a FIR system (no feedback) and the overall system function is as given above.

(b) Is your answer to part (a) unique? Explain.

(c) Plot the poles and zeros of $H(z)$ in the z-plane and sketch the magnitude of the overall frequency response for $-\pi < \hat{\omega} < \pi$.

(d) If the input is $x[n] = e^{j\hat{\omega}n}$, for what values of $\hat{\omega}$ will $y[n] = 0$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a) $H_1(z) = (1-z^{-1})(1+z^{-2}) = 1 - z^{-1} + z^{-2} - z^{-3}$

$H_2(z) = \dfrac{1}{1+.8z^{-1}}$

(b) NOT UNIQUE.

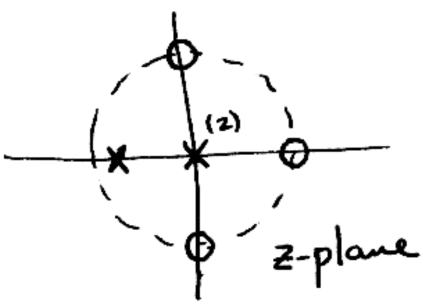
Another answer is: $H_1(z) = 1 + z^{-2}$ $\quad H_2(z) = \dfrac{1-z^{-1}}{1+.8z^{-1}}$

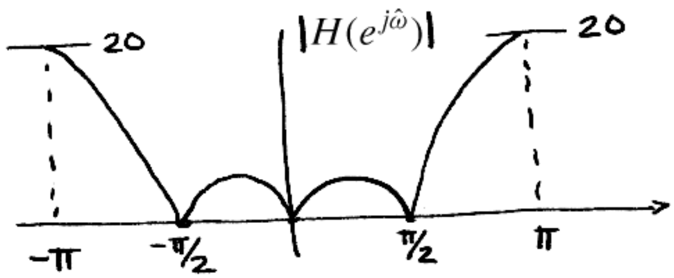
(c) $H(z) = \dfrac{(1-z^{-1})(1+z^{-2})}{1+.8z^{-1}} = \dfrac{(z-1)(z^2+1)}{z^2(z+.8)}$

POLES @ $z = -0.8$ and $z = 0$ (TWO)

ZEROS @ $z = 1$ and $z = \pm j$

$z^2+1=0 \Rightarrow z^2 = -1$
$\Rightarrow z = \sqrt{-1}$





At $\hat{\omega} = \pi \Rightarrow z = -1$

$H(-1) = \dfrac{(1-(-1))(1+1)}{1-.8} = \dfrac{(2)(2)}{.2} = 20$

(d) If $x[n] = e^{j\hat{\omega}n}$ then $y[n] = H(e^{j\hat{\omega}})e^{j\hat{\omega}n}$

To get $y[n] = 0$ we need $H(e^{j\hat{\omega}}) = 0$

This happens at $\hat{\omega} = -\frac{\pi}{2}, 0, \frac{\pi}{2}$