PROBLEM:

A linear time-invariant system has impulse response

$$h[n] = (0.9)^n u[n]$$

(a) What is the frequency response of the system?

(b) The input to this system is

$$x[n] = 5 - 4\delta[n-2] + 10\cos(0.5\pi n + \pi/3)$$

Determine an equation for the output of the system $y[n]$ corresponding to the above input $x[n]$. Give an equation for $y[n]$ that is valid for all n. (*Hint: Use superposition to make this an easy problem.*

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) $h[n] = (0.9)^n u[n] = \begin{cases} (0.9)^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$

This is the impulse response of the following difference equation:

$$y[n] = 0.9y[n] + x[n]$$

$$\Rightarrow H(z) = \frac{1}{1-0.9z^{-1}} \quad \Rightarrow \quad H(e^{j\hat{\omega}}) = \frac{1}{1-0.9e^{-j\hat{\omega}}}$$

(b) $x[n]$ is the sum of three signals: $x_1[n] + x_2[n] + x_3[n]$

$x_1[n] = 5$ is $5e^{j0n}$ (zero freq).

$$y_1[n] = H(e^{j0})x_1[n] = \frac{1}{1-.9}(5) = \frac{5}{.1} = 50$$

$x_2[n] = -4\,\delta[n-2]$. (shifted impulse)

$$y_2[n] = -4h[n-2] = -4(0.9)^{n-2}u[n-2]$$ ← STARTS AT $n=2$

$x_3[n] = 10\cos(0.5\pi n + \pi/3)$ (freq $= 0.5\pi$)

$$H(e^{j\pi/2}) = \frac{1}{1-.9e^{-j\pi/2}} = \frac{1}{1-.9j} = 0.743e^{j0.233\pi}$$

$$\Rightarrow y_3[n] = 7.43\cos(0.5\pi n + 0.566\pi)$$

$$y[n] = y_1[n] + y_2[n] + y_3[n]$$

$$= 50 - 4(0.9)^{n-2}u[n-2] + 7.43\cos(0.5\pi n + 0.566\pi)$$