



## PROBLEM:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] + x[n - 4]$$

- Find its impulse response  $h[n]$ .
- Find its system function  $H(z)$ .
- Plot the poles and zeros of  $H(z)$  in the  $z$ -plane.  
(Recall that  $-1 = e^{j\pi k}$  where  $k$  is an *odd* integer.)
- Find the frequency response  $H(e^{j\hat{\omega}})$  and express it in polar form (magnitude and phase).
- Carefully sketch and label a plot of  $|H(e^{j\hat{\omega}})|$  for  $-\pi < \hat{\omega} < \pi$ .

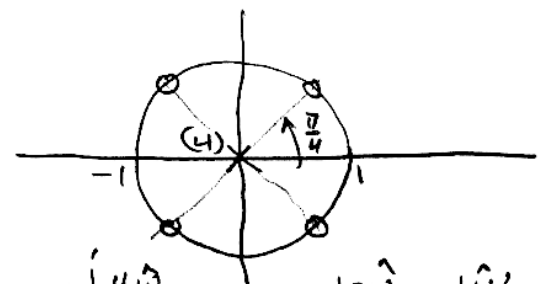


$$y[n] = x[n] + x[n-4]$$

a)  $h[n] = \delta[n] + \delta[n-4]$

(b)  $H(z) = 1 + z^{-4} = (z^4 + 1)/z^4$

(c) zeros:  $z^4 = -1 = e^{j\pi k}$   
 $\Rightarrow z_k = e^{j\frac{\pi k}{4}}$   $k$  odd



(d)  $H(e^{j\hat{\omega}}) = H(z) \Big|_{z=e^{j\hat{\omega}}} = 1 + e^{-j4\hat{\omega}} = e^{-j2\hat{\omega}} (e^{j2\hat{\omega}} + e^{-j2\hat{\omega}})$   
 $= 2 \cos(2\hat{\omega}) e^{-j2\hat{\omega}}$

$$|H(e^{j\hat{\omega}})| = |2 \cos(2\hat{\omega})|$$

$$\angle H(e^{j\hat{\omega}}) = -2\hat{\omega} \pm \pi \text{ when } \cos 2\hat{\omega} < 0$$

