PROBLEM:

Circle the correct answer to each of these short answer questions (3 pts. each):

1. A particular system may be viewed as a cascade of two systems whose separate system functions are $H_1(z) = 1 - z^{-1}$ and $H_2(z) = 1 + z^{-2}$. Determine $H(z)$, the overall system function.

   (a) $H(z) = 1 + z^{-1} + z^{-2} + z^{-3}$

   (b) $H(z) = 1 - z^{-1} + z^{-2}$

   (c) $H(z) = \dfrac{1 - z^{-1}}{1 + z^{-2}}$

   (d) $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

2. Pick the correct frequency response for the FIR filter: $y[n] = x[n] - x[n-1]$

   (a) $\delta[n] - \delta[n-1]$

   (b) $\sin(\frac{1}{2}\hat{\omega})$

   (c) $2e^{-j(\hat{\omega}-\pi)/2}\sin(\frac{1}{2}\hat{\omega})$

   (d) $|2\sin(\frac{1}{2}\hat{\omega})|$

   (e) none of the above

3. If $H(z) = z^{-3}$, the filter has a frequency response (magnitude) that is:

   (a) constant for all $\hat{\omega}$

   (b) a lowpass filter

   (c) a highpass filter

   (d) a bandpass filter

   (e) equal to $\delta[n-3]$

4. If $H(z) = \dfrac{z^{-3}}{1 - 0.75z^{-1}}$, the value of the frequency response at $\hat{\omega} = \frac{1}{2}\pi$ is equal to

   (a) zero

   (b) $0.8e^{j0.295\pi}$

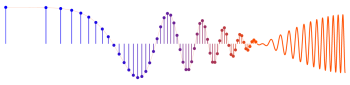
   (c) $0.8e^{-j0.295\pi}$

   (d) $0.2e^{j0.5\pi}$

   (e) 4

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



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   (c) $H(z) = \dfrac{1 - z^{-1}}{1 + z^{-2}}$
   (d) $H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$

$$H_1(z)H_2(z) = (1 - z^{-1})(1 + z^{-2})$$
$$= 1 - z^{-1} + z^{-2} - z^{-3}$$

2. Pick the correct frequency response for the FIR filter: $y[n] = x[n] - x[n-1]$

   (a) $\delta[n] - \delta[n-1]$
   (b) $\sin(\frac{1}{2}\hat{\omega})$
   (c) $2e^{-j(\hat{\omega}-\pi)/2}\sin(\frac{1}{2}\hat{\omega})$
   (d) $|2\sin(\frac{1}{2}\hat{\omega})|$
   (e) none of the above

$$H(e^{j\hat{\omega}}) = 1 - e^{-j\hat{\omega}} = e^{-j\hat{\omega}/2}\left(e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}\right)$$
$$= 2je^{-j\hat{\omega}/2}\sin(\hat{\omega}/2)$$
$$= 2e^{-j(\hat{\omega}/2 - \pi/2)}\sin(\hat{\omega}/2)$$

3. If $H(z) = z^{-3}$, the filter has a frequency response (magnitude) that is:

   (a) constant for all $\hat{\omega}$
   (b) a lowpass filter
   (c) a highpass filter
   (d) a bandpass filter
   (e) equal to $\delta[n-3]$

$$H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}$$
$$|H(e^{j\hat{\omega}})| = 1$$

4. If $H(z) = \dfrac{z^{-3}}{1 - 0.75z^{-1}}$, the value of the frequency response at $\hat{\omega} = \frac{1}{2}\pi$ is equal to

   (a) zero
   (b) $0.8e^{j0.295\pi}$
   (c) $0.8e^{-j0.295\pi}$
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   (e) 4

$$\hat{\omega} = \tfrac{1}{2}\pi \Rightarrow z = e^{j\hat{\omega}} = e^{j\pi/2} = j$$
$$H(e^{j\pi/2}) = \frac{j^{-3}}{1 - \frac{3}{4}j^{-1}} = \frac{j4}{4 + 3j}$$
$$\cong \frac{4e^{j\pi/2}}{5e^{j0.2\pi}} = 0.8e^{j0.3\pi}$$