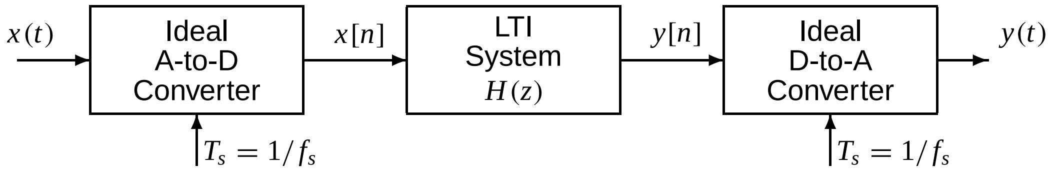
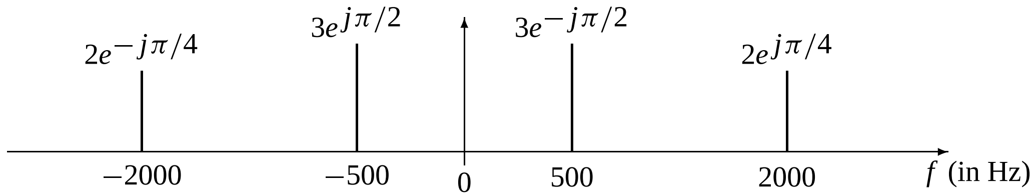
PROBLEM:

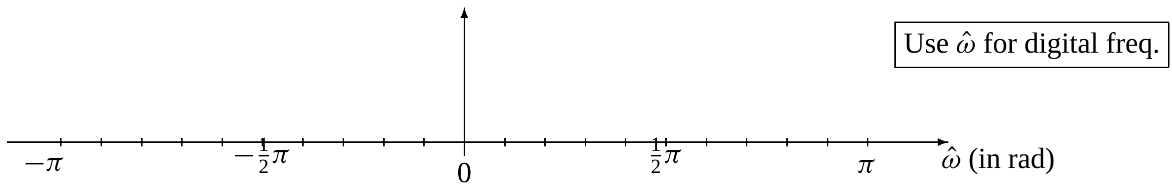
The input to the A-to-D converter in the figure below consists of sinusoids defined by a spectrum plot. The system function for the LTI system is a digital FIR filter. Since digital filters can be used to null out individual sinusoids, it should be possible to design $H(z)$ so that all the frequency components *will be zeroed out.*



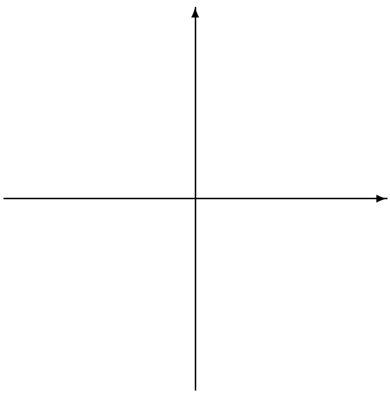
(a) If the input $x(t)$ is given by the two-sided spectrum representation shown below,



Determine the spectrum for $x[n]$ when $f_s = 2000$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, but label the frequency, amplitude and phase of each spectral component.



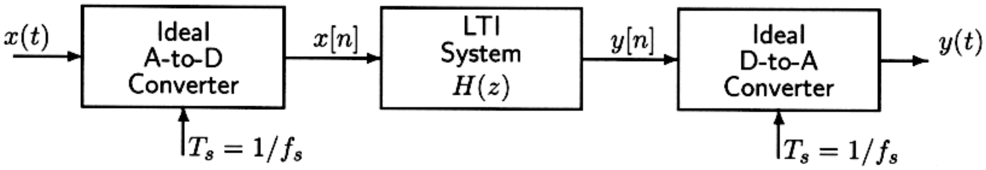
(b) Now you must design the FIR filter: $H(z) = \sum_{k=0}^{M} b_k z^{-k}$. To avoid the all zero solution, set $b_0 = 1$. If the objective is to make the output zero by filtering $x[n]$, then the FIR filter $H(z)$ must be determined by specifying the locations of its zeros in *either* the z domain or the $\hat{\omega}$ domain. Draw the pole-zero diagram for $H(z)$.



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

The input to the A-to-D converter in the figure below consists of sinusoids defined by a spectrum plot. The system function for the LTI system is a digital FIR filter. Since digital filters can be used to null out individual sinusoids, it should be possible to design $H(z)$ so that all the frequency components *will be zeroed out.*



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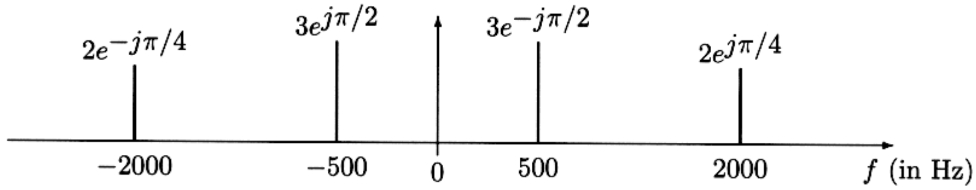
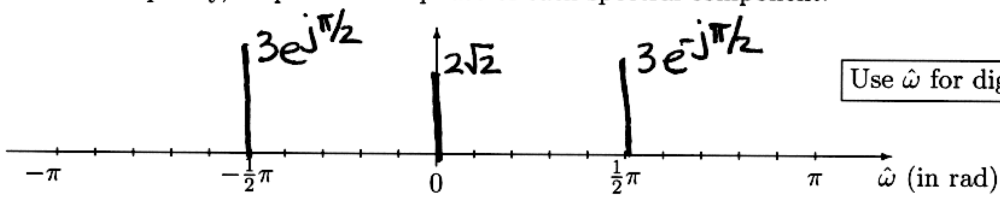


Determine the spectrum for $x[n]$ when $f_s = 2000$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, but label the frequency, amplitude and phase of each spectral component.



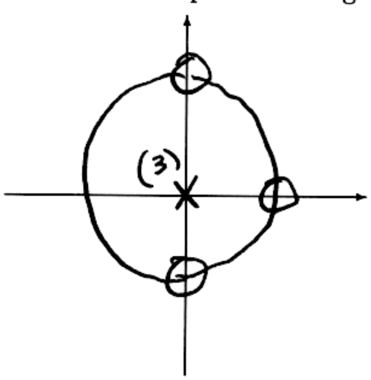
Use $\hat{\omega}$ for digital freq.

$f = 2000\text{Hz} \Rightarrow \hat{\omega} = 2\pi f/f_s = 2\pi(2000)/2000 = 2\pi \longrightarrow 0$

At $\hat{\omega} = 0$ we Add $2e^{j\pi/4} + 2e^{-j\pi/4} = 2 \cdot 2\cos(\pi/4) = 2\sqrt{2}$

$f = 500\text{Hz} \Rightarrow \hat{\omega} = 2\pi(500)/2000 = 2\pi/4 = \pi/2$

(b) Now you must design the FIR filter: $H(z) = \sum_{k=0}^{M} b_k z^k$. To avoid the all zero solution, set $b_0 = 1$. If the objective is to make the output zero by filtering $x[n]$, then the FIR filter $H(z)$ must be determined by specifying the locations of its zeros in *either* the z domain or the $\hat{\omega}$ domain. Draw the pole-zero diagram for $H(z)$.



Need zeros at
$\hat{\omega} = 0 \quad \hat{\omega} = \pm\pi/2$
$\Rightarrow z = 1 \quad z = \pm j$

There would be 3 poles at $z = 0$, but they don't have a significant effect on the frequency response

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