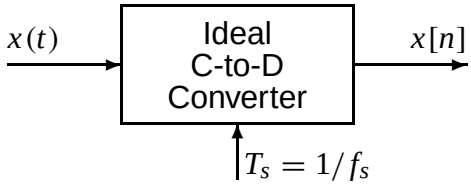
PROBLEM:



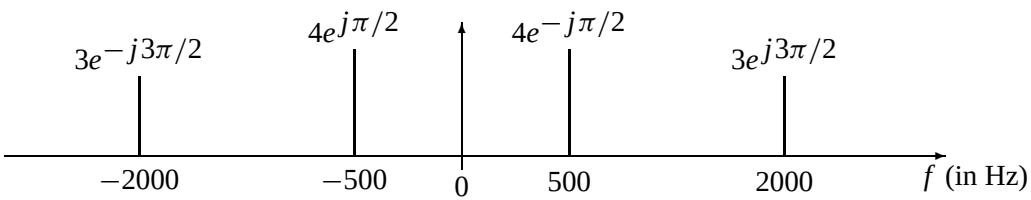
(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 10\cos(0.20\pi n - \pi/3)$$

If the sampling rate is $f_s = 2500$ samples/second, many *different* continuous-time signals $x(t) = x_i(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 2500 Hz; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/2500$. Give *only* the frequency and phase for each signal.

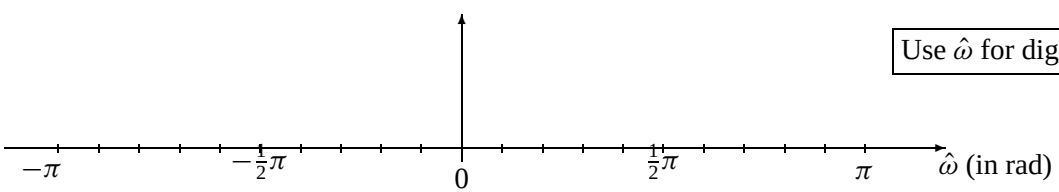
$x_1(t)$:	$f_1 =$ Hz	$\phi_1 =$ rads
$x_2(t)$:	$f_2 =$ Hz	$\phi_2 =$ rads

(b) If the input $x(t)$ is given by the two-sided spectrum representation shown below,



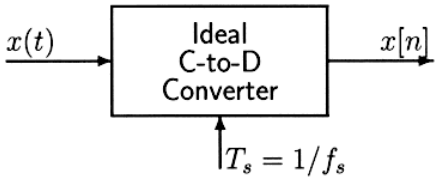
Determine the spectrum for $x[n]$ when $f_s = 2500$ samples/sec. Make a plot for your answer, but label the frequency, amplitude and phase of each spectral component.

Use $\hat{\omega}$ for digital freq.



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



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$x_1(t)$: $f_1 = 250$ Hz | $\phi_1 = -\pi/3$ rads

$x_2(t)$: $f_2 = 2250$ Hz | $\phi_2 = \pi/3$ rads

$t = n/f_s \Rightarrow n = f_s t = 2500t$

$x_1(t) = 10\cos(0.2\pi(2500)t - \pi/3)$

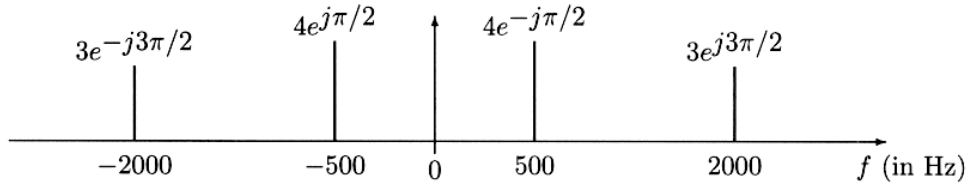
$= 10\cos(2\pi(250)t - \pi/3)$

Can add or subtract f_s:

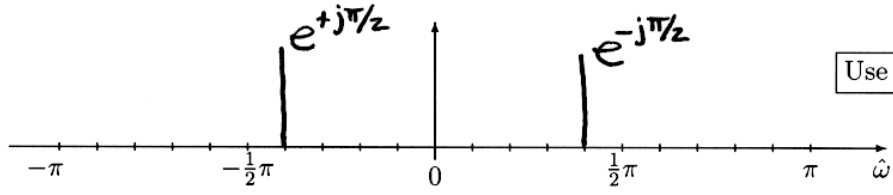
$x_2(t) = 10\cos(2\pi(250 - 2500)t - \pi/3)$

$= 10\cos(2\pi(-2250)t - \pi/3) = 10\cos(2\pi(2250)t + \pi/3)$

(b) If the input $x(t)$ is given by the two-sided spectrum representation shown below,



Determine the spectrum for $x[n]$ when $f_s = 2500$ samples/sec. Make a plot for your answer, but label the frequency, amplitude and phase of each spectral component.



Use $\hat{\omega}$ for digital freq.

$\hat{\omega} = 2\pi f/f_s = 2\pi(f/f_s) \Rightarrow f = 500 \rightarrow \hat{\omega} = 2\pi\left(\frac{500}{2500}\right) = 2\pi/5$

$f = -500 \rightarrow \hat{\omega} = -2\pi/5$

$f = 2000 \rightarrow \hat{\omega} = 2\pi\left(\frac{2000}{2500}\right) = 2\pi(4/5)$

same as $\hat{\omega} - 2\pi = 2\pi(4/5 - 1) = -2\pi/5$

$f = -2000 \rightarrow \hat{\omega} = +2\pi/5$

$4e^{-j\pi/2} + 3e^{-j3\pi/2} = 1e^{-j\pi/2}$

$\therefore$ we must add the $f = +500$ Hz and the $f = -2000$ Hz components