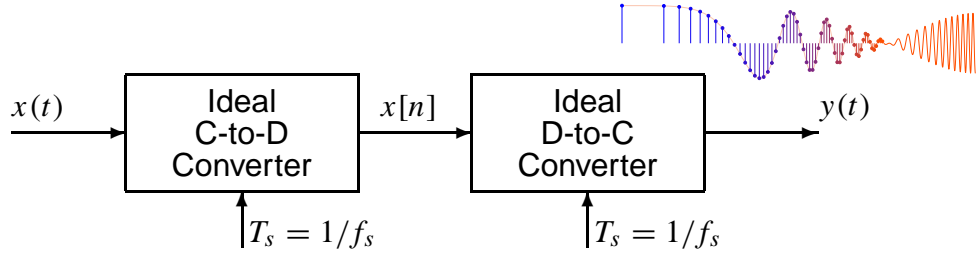


PROBLEM:



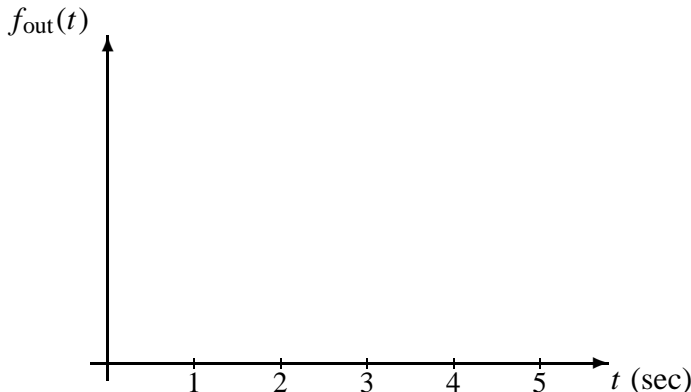
- (a) A continuous-time signal $x(t)$ is defined by the following formula: $x(t) = \sum_{k=-8}^8 \frac{j\pi k}{k^2 + 1} e^{j20\pi kt}$ Determine the Nyquist rate for sampling $x(t)$; give your answer in Hz or in samples per second.

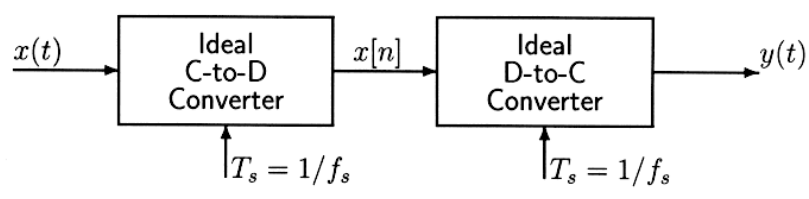
$$f_s = \quad \text{Hz}$$

- (b) A chirp signal is defined as follows:

$$x(t) = \cos(400\pi t^2) \quad \text{for } 0 \leq t \leq 5 \text{ sec.}$$

If the sampling rate is $f_s = 800$ Hz, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction**. Hint: this is similar to the laboratory exercise of putting the sampled chirp signal into the MATLAB function `specgram()`.





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Determine the Nyquist rate for sampling $x(t)$; give your answer in Hz or in samples per second.

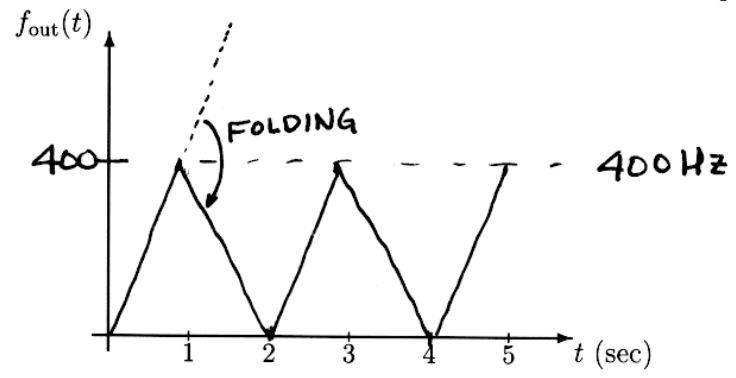
$f_s = 160 \text{ Hz}$

Highest frequency in $x(t)$ is $\omega_k = \omega_8$
 $\omega_{MAX} = (20\pi)(8) = 160\pi$
 $f_{MAX} = 80 \text{ Hz}$
 $f_s \geq 2f_{MAX} = 160 \text{ Hz}$

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If the sampling rate is $f_s = 800 \text{ Hz}$, then the output signal $y(t)$ will have time-varying frequency content. Draw a graph of the resulting analog *instantaneous* frequency (in Hz) versus time of the signal $y(t)$ **after reconstruction**. Hint: this is similar to the laboratory exercise of putting the sampled chirp signal into the MATLAB function `spectrogram()`.



$\psi(t) = 2\pi(200t^2)$

$f_i(t) = \frac{1}{2\pi} \frac{d}{dt} \psi(t) = \frac{1}{2\pi} (2\pi)(400t) = 400t \text{ Hz}$

$f_s = 800 \text{ Hz} \Rightarrow$ D/A cannot reconstruct anything above 400 Hz. Alias/Fold at 400 Hz

$f_i(0) = 0$; $f_i(1) = 400$; $f_i(2) = 800$ $f_i(3) = 1200 \text{ Hz}$
↑ alias to $f = 0$ ↑ alias to 400 Hz