PROBLEM:

A discrete-time system is defined by the following system function:

$$H(z) = \frac{0.81 + z^{-2}}{1 + 0.81z^{-2}}.$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

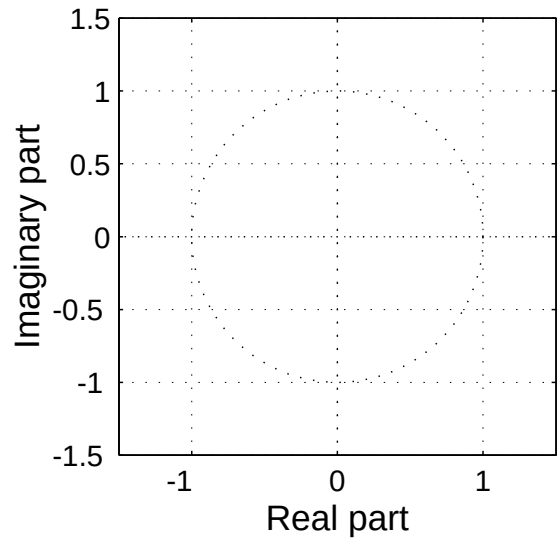
(b) Fill in numbers for the vectors `bb` and `aa` in the following MATLAB computation of the frequency response of the system:

```
bb=[                    ];     aa=[                    ];

yy=filter(bb,aa,xx)
```

where `xx` is the input signal to be filtered.

(c) Determine *all* the poles and zeros of $H(z)$ and plot them in the z-plane.



(d) Compute $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$, the magnitude-squared of the frequency response. *Your answer should only contain real quantities.*

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
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SOLUTION

A discrete-time system is defined by the following system function:

$$H(z) = \frac{0.81 + z^{-2}}{1 + 0.81z^{-2}}.$$

(a) Write down the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of the system.

$$y[n] = -0.81\,y[n-2] + 0.81\,x[n] + x[n-2]$$

(b) Fill in numbers for the vectors `bb` and `aa` in the following MATLAB computation of the frequency response of the system:

```
bb=[0.81, 0, 1 ];    aa=[ 1, 0, .81 ];

yy=filter(bb,aa,xx)
```

where `xx` is the input signal to be filtered.

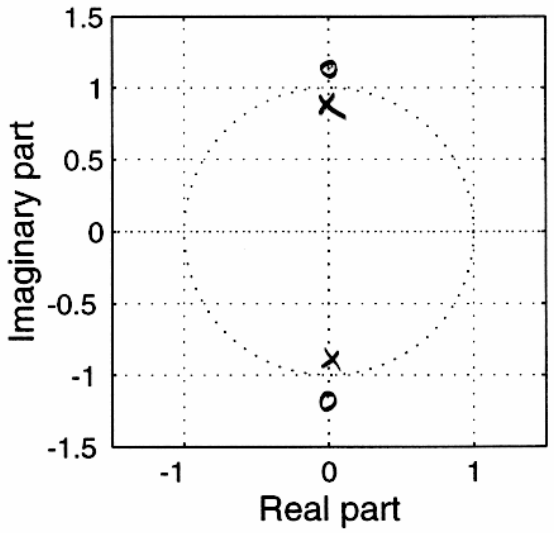
(c) Determine *all* the poles and zeros of $H(z)$ and plot them in the z-plane.

$$H(z) = \frac{0.81z^2 + 1}{z^2 + .81}$$

$$= \frac{.81\left(z - j\frac{1}{.9}\right)\left(z + j\frac{1}{.9}\right)}{(z - j.9)(z + j.9)}$$

Zeros at $z = \pm \frac{10}{9}j$

Poles at $z = \pm .9j$



(d) Compute $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$, the magnitude-squared of the frequency response.

$$|H(e^{j\hat{\omega}})|^2 = \left(\frac{.81 + e^{-j2\hat{\omega}}}{1 + .81e^{-j2\hat{\omega}}}\right)\left(\frac{.81 + e^{j2\hat{\omega}}}{1 + .81e^{j2\hat{\omega}}}\right)$$

$$= \frac{(.81)^2 + 1.62\cos 2\hat{\omega} + 1}{1 + 1.62\cos 2\hat{\omega} + (.81)^2} = 1$$

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