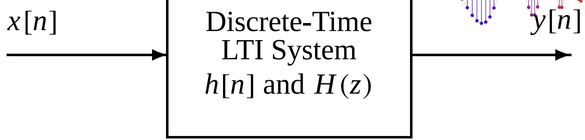
PROBLEM:



An LTI discrete-time system is depicted above. The system function of the system is

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 + \frac{3}{4}z^{-1})}.$$

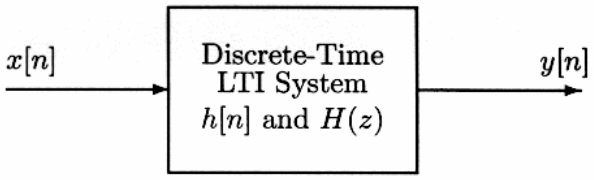
(a) It is desired that the *output* of the system be $y[n] = (\frac{1}{2})^n u[n]$. Find the z-transform $Y(z)$ of this output signal.

(b) Use the z-transform method to determine the z-transform $X(z)$ of the input to the system such that the output of the system will be $y[n] = (\frac{1}{2})^n u[n]$.

(c) Use the partial fraction expansion method to determine the impulse response $h[n]$ of the system with system function $H(z)$ given above.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION



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(a) It is desired that the *output* of the system be $y[n] = (\frac{1}{2})^n u[n]$. Find the z-transform $Y(z)$ of this output signal.

$$Y(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

(b) Use the z-transform method to determine the z-transform $X(z)$ of the input to the system such that the output of the system will be $y[n] = (\frac{1}{2})^n u[n]$.

$$Y(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} = H(z)\,X(z)$$

$$\therefore X(z) = \frac{\dfrac{1}{1 - \frac{1}{2}z^{-1}}}{\dfrac{1}{(1 - \frac{1}{2}z^{-1})(1 + \frac{3}{4}z^{-1})}} = 1 + \frac{3}{4}z^{-1}$$

(c) Use the partial fraction expansion method to determine the impulse response $h[n]$ of the system with system function $H(z)$ given above.

$$H(z) = \frac{\dfrac{1}{1 + \frac{1}{2}\cdot\frac{4}{3}}}{1 + \frac{3}{4}z^{-1}} + \frac{\dfrac{1}{1 + \frac{3}{4}\cdot 2}}{1 - \frac{1}{2}z^{-1}}$$

$$= \frac{\frac{3}{5}}{1 + \frac{3}{4}z^{-1}} + \frac{\frac{2}{5}}{1 - \frac{1}{2}z^{-1}}$$

$$h[n] = \frac{3}{5}\left(-\frac{3}{4}\right)^n u[n] + \frac{2}{5}\left(\frac{1}{2}\right)^n u[n]$$

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