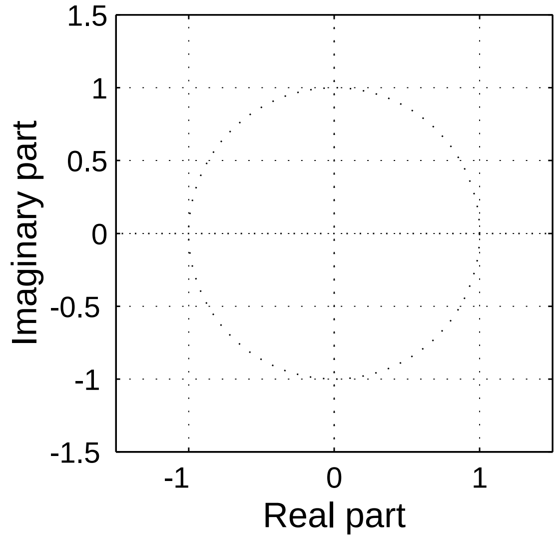
## PROBLEM:

A discrete-time system is defined by the following system function:

$$H(z) = \frac{1 + z^{-2}}{1 - 0.75z^{-1}} = \frac{1}{1 - 0.75z^{-1}} + \frac{z^{-2}}{1 - 0.75z^{-1}}.$$

(a) Use the first form of $H(z)$ to determine *all* the poles and zeros of $H(z)$ and plot them in the $z$-plane.




(b) Use the second form of $H(z)$ above to find the corresponding impulse response $h[n]$.

(c) Use the first form of $H(z)$ to obtain an expression for the magnitude-squared of the frequency response $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$. Your answer should involve only real quantities.

(d) For what value (or values) of $\hat{\omega}$ will it be true that $y[n] = 0$ for $-\infty < n < \infty$ when the input to the system is $x[n] = e^{j\hat{\omega}n}$ for $-\infty < n < \infty$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.


SOLUTION

A discrete-time system is defined by the following system function:

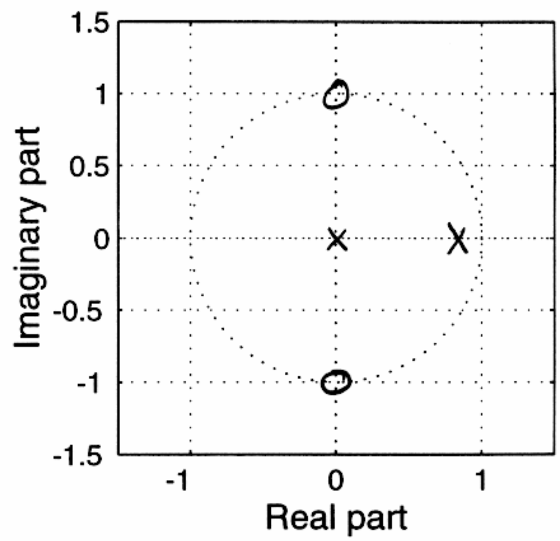
$$H(z) = \frac{1+z^{-2}}{1-0.75z^{-1}} = \frac{1}{1-0.75z^{-1}} + \frac{z^{-2}}{1-0.75z^{-1}}.$$

(a) Use the first form of $H(z)$ to determine *all* the poles and zeros of $H(z)$ and plot them in the $z$-plane.

$$H(z) = \frac{z^2+1}{z(z-.75)} = \frac{(z-j)(z+j)}{z(z-.75)}$$

zeros at $z = \pm j$

Poles at $z = 0, .75$




(b) Use the second form of $H(z)$ above to find the corresponding impulse response $h[n]$.

$$h[n] = (.75)^n u[n] + (.75)^{n-2} u[n-2]$$

(c) Use the first form of $H(z)$ to obtain an expression for the magnitude-squared of the frequency response $|H(e^{j\hat{\omega}})|^2 = H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}})$. Your answer should involve only real quantities.

$$|H(e^{j\hat{\omega}})|^2 = \left(\frac{1+e^{-j2\hat{\omega}}}{1-.75e^{-j\hat{\omega}}}\right)\left(\frac{1+e^{j2\hat{\omega}}}{1-.75e^{j\hat{\omega}}}\right)$$

$$= \frac{1+2\cos(2\hat{\omega})+1}{1-1{,}5\cos\hat{\omega}+(.75)^2} = \frac{2(1+\cos(2\hat{\omega}))}{1{,}5625-1{,}5\cos\hat{\omega}}$$

(d) For what value of $\hat{\omega}$ will it be true that $y[n] = 0$ for $-\infty < n < \infty$ when the input to the system is $x[n] = e^{j\hat{\omega}n}$ for $-\infty < n < \infty$?

Since we have a zero at $z = \pm j = e^{j\pm\pi/2}$

$$\hat{\omega} = \pm \pi/2$$

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