PROBLEM:

Suppose that a MATLAB function has been written to calculate a sum of discrete-time sinusoids, e.g., something similar to the `sinus( )` that was written for the lab. Here is the actual function:

```
function  xn = makedcos(omegahat,XX,Length)
xn = real( exp( j*(0:Length-1)’*omegahat(:)’ ) * XX(:));
```

(a) Write an equation for $x[n]$, the discrete-time signal that is created by this MATLAB function, when the following function call is used:

```
x = makedcos(pi*[0,0.5,0.8,1.2],[1,-2i,1i,1-1i],100001)
```

Your equation should be in terms of cosine functions. To do this you must figure out how the matrix multiplications and `exp( )` in the MATLAB statement defining `xn` work.

(b) Draw a plot of the discrete-time spectrum (vs. $\hat{\omega}$) of the discrete-time signal defined by this MATLAB operation. Make sure that you include all the spectrum components in the $-\pi$ to $+\pi$ interval.

(c) The following MATLAB commands are used to make an output sound:

```
x = makedcos(pi*[0,0.5,0.8,1.2],[1,-2i,1i,1-1i],100001)
soundsc(x,2000)
```

Draw a plot of the (idealized) continuous-time spectrum (vs. f in Hz) of the continuous-time signal $x(t)$ that would be created at the output of an ideal D-to-C converter (approximately realized by the `soundsc()` function).

(d) Write an equation for $x(t)$, the continuous-time signal that is created at the output of the ideal D-to-C converter.

(e) What is the duration (in seconds) of the continuous-time signal $x(t)$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

(a) We have four frequencies of $\hat{\omega}_1 = 0$, $\hat{\omega}_2 = 0.5\pi$, $\hat{\omega}_3 = 0.8\pi$, and $\hat{\omega}_4 = 1.2\pi$, and the corresponding complex amplitudes are $X_1 = 1$, $X_2 = -j2$, $X_3 = j$, and $X_4 = 1 - j = \sqrt{2}e^{-j\pi/4}$. The `makedcos()` function implements the following mathematical equation:

$$x[n] = \sum_{k=1}^{N} \Re e\left\{X_k e^{j\hat{\omega}_k n}\right\} = \sum_{k=1}^{N} A_k \cos(\hat{\omega}_k + \phi_k)$$

where $A_k = |X_k|$ and $\phi_k = \angle X_k$.

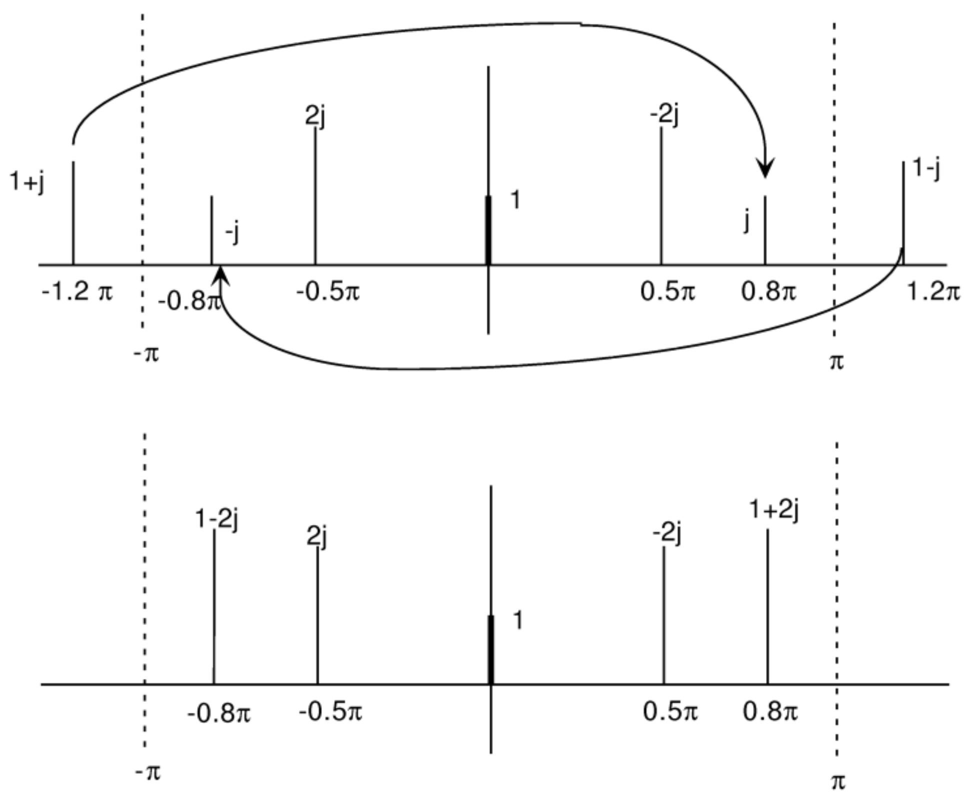
Now for some *folding*: since $\cos(1.2\pi n - \pi/4) = \cos(-0.8\pi n - \pi/4) = \cos(0.8\pi n + \pi/4)$, we get

$$\begin{aligned} x[n] &= 1 + 2\cos(0.5\pi n - \pi/2) + \cos(0.8\pi n + \pi/2) + \sqrt{2}\cos(1.2\pi n - \pi/4) \\ &= 1 + 2\cos(0.5\pi n - \pi/2) + \cos(0.8\pi n + \pi/2) + \sqrt{2}\cos(0.8\pi n + \pi/4) \\ &= 1 + 2\cos(0.5\pi n - \pi/2) + \sqrt{5}\cos(0.8\pi n + 0.352\pi) \end{aligned}$$

where the last line results from a "phasor addition" of the two cosines that both have a frequency of $\hat{\omega} = 0.8\pi$ (after the folding). We must add X_3 and the conjugate of X_4 because X_4^* was the complex amplitude at $\hat{\omega} = -1.2\pi$ which is then mapped to $\hat{\omega} = 0.8\pi$ because we add 2π to $\hat{\omega} = -1.2\pi$. Thus, we add complex amplitudes and convert to polar form:

$$X_3 + X_4^* = j + (1 - j)^* = j + 1 + j = 1 + j2 = \sqrt{5}e^{j0.352\pi}$$

(b) see plot

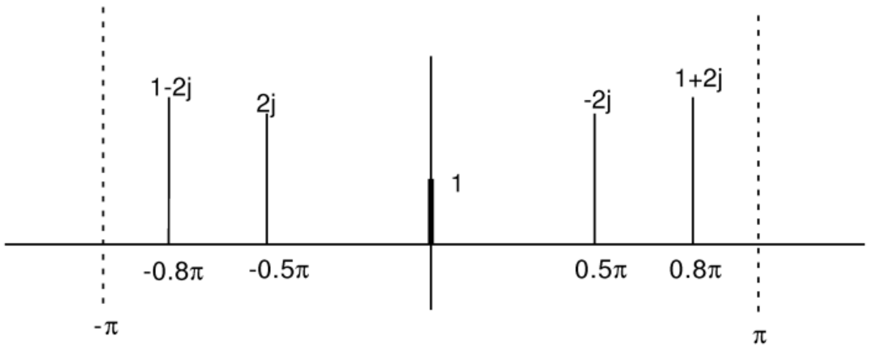


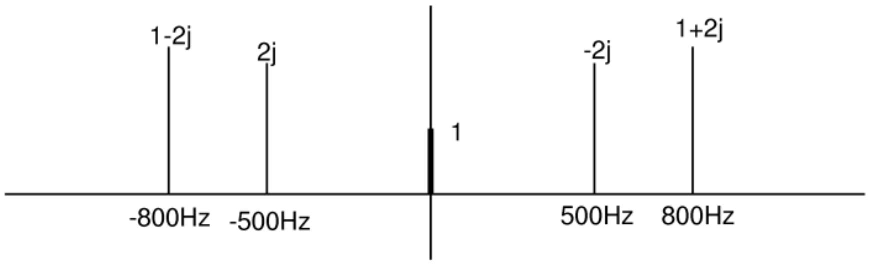
McClellan, Schafer, and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

(c) Sampling rate is $f_s = 2000$ Hz. We have to convert the $\hat{\omega}$ frequencies back to continuous-time frequencies in hertz, using the conversion:

$$f = \frac{\hat{\omega}}{2\pi} f_s$$

Thus we get: $f_1 = 0$ which is the DC component, $f_2 = (0.5\pi/2\pi)(2000)$ Hz = 500 Hz, and $f_3 = f_4 = (0.8\pi/2\pi)(2000)$ Hz = 800 Hz.





(d) The formula for the continuous-time output signal $x(t)$ has the same magnitudes and phases as $x[n]$, just different frequencies:

$$x(t) \;=\; 1 + 2\cos(2\pi(500)t - \pi/2) + \sqrt{5}\cos(2\pi(800)t + 0.352\pi)$$

Notice that the output signal contains no frequencies greater than 1000 Hz which is half the sampling frequency.

(e) Number of Samples = 100001, but we can say that the range of n is $n = 0$ to $n = 100,000$. Since the sampling rate = 2000 samples/sec., we can convert n to t via the equation:

$$t = n/f_s$$

Thus $x(t)$ starts at $t = 0$ and ends at $t = (100,000)/f_s$, and the duration is

$$\frac{100000 \text{ samples}}{2000 \text{ samples/sec}} = 50\,s$$