PROBLEM:

We now have four ways of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = \frac{1}{4}(x[n] - x[n-4])$.

(b) $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$.

(c) $H(e^{j\hat{\omega}}) = [2 + 2\cos(\hat{\omega})]e^{-j\hat{\omega}2}$.

(d) $H(z) = z^{-3} + z^{-6} + z^{-9}$.

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SOLUTION

We now have four ways of describing an LTI system: the difference equation; the impulse response, $h[n]$; the frequency response, $H(e^{j\hat{\omega}})$; and the system function, $H(z)$. In the following, you are given one of these representations and you must find the other three.

(a) $y[n] = \frac{1}{4}(x[n] - x[n-4])$.

$$h[n] = \tfrac{1}{4}\left(\delta[n] - \delta[n-4]\right)$$

$$H(z) = \tfrac{1}{4}\left(1 - z^{-4}\right)$$

$$H(e^{j\hat{\omega}}) = \tfrac{1}{4}\left(1 - e^{-j4\hat{\omega}}\right) = \tfrac{1}{4}e^{-j2\hat{\omega}}(e^{j2\hat{\omega}} - e^{-j2\hat{\omega}}) = \frac{j}{2}\sin(2\hat{\omega})e^{-j2\hat{\omega}}$$

(b) $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]$.

$$y[n] = x[n] + 2x[n-1] + 3x[n-2] + 2x[n-3] + x[n-4]$$

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$$

$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + 3e^{-j2\hat{\omega}} + 2e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} \\ &= e^{-j2\hat{\omega}}\left(3 + 2e^{j\hat{\omega}} + 2e^{-j\hat{\omega}} + e^{j2\hat{\omega}} + e^{-j2\hat{\omega}}\right) \\ &= e^{-j2\hat{\omega}}\left(3 + 4\cos(\hat{\omega}) + \cos(2\hat{\omega})\right) \end{aligned}$$

$$\text{or,}\quad H(e^{j\hat{\omega}}) = \frac{1 - e^{-j5\hat{\omega}}}{1 - e^{-j\hat{\omega}}} = \frac{e^{-j2.5\hat{\omega}}(e^{j2.5\hat{\omega}} - e^{-j2.5\hat{\omega}})}{e^{-j0.5\hat{\omega}}(e^{j0.5\hat{\omega}} - e^{-j0.5\hat{\omega}})} = e^{-j2\hat{\omega}}\left(\frac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})}\right)$$

(c) $H(e^{j\hat{\omega}}) = [2 + 2\cos(\hat{\omega})]e^{-j\hat{\omega}2}$.

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}$$

$$h[n] = \delta[n-1] + 2\delta[n-2] + \delta[n-3]$$

$$y[n] = x[n-1] + 2x[n-2] + x[n-3]$$

$$H(z) = z^{-1} + 2z^{-2} + z^{-3}$$

(d) $H(z) = z^{-3} + z^{-6} + z^{-9}$.

$$y[n] = x[n-3] + x[n-6] + x[n-3]$$

$$h[n] = \delta[n-3] + \delta[n-6] + \delta[n-3]$$

$$H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}} + e^{-j6\hat{\omega}} + e^{-j9\hat{\omega}} = e^{-j6\hat{\omega}}\left(1 + 2\cos(3\hat{\omega})\right)$$

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