PROBLEM:

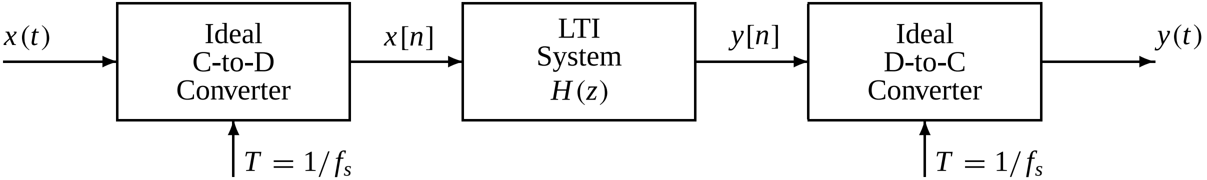
The input to the C-to-D converter in the figure below is

$$x(t) = 3 + 2\cos(6000\pi t - \pi/4) + 11\cos(12000\pi t - \pi/3)$$

The system function for the LTI system is

$$H(z) = \frac{1}{4}(1 - z^{-4})$$

If $f_s = 8000$ samples/second, determine an expression for $y(t)$, the output of the D-to-C converter.



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

The input to the C-to-D converter in the figure below is

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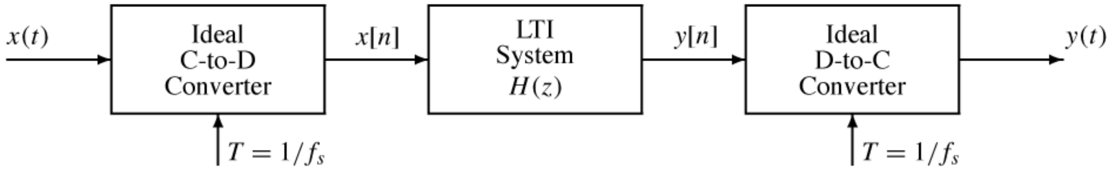

$$H(z) = \frac{1}{4}\left(1 - z^{-1}\right)$$

$$H(e^{j\hat{\omega}}) = \tfrac{1}{4}(1 - e^{-j4\hat{\omega}}) = \tfrac{1}{4}(e^{j2\hat{\omega}} - e^{-j2\hat{\omega}})e^{-j2\hat{\omega}} = \frac{j}{2}\sin(2\hat{\omega})e^{-j2\hat{\omega}} = \tfrac{1}{2}\sin(2\hat{\omega})e^{j(\pi/2)}e^{-j2\hat{\omega}}$$

Since $f_s = 8000$ samples / second, we get the discrete-time cosines for $x[n]$:

$$\begin{aligned} x[n] &= 3 + 2\cos(\tfrac{3}{4}\pi n - \pi/4) + 11\cos(\tfrac{6}{4}\pi n - \pi/3) \\ &= 3 + 2\cos(\tfrac{3}{4}\pi n - \pi/4) + 11\cos(-\tfrac{2}{4}\pi n - \pi/3) \\ &= 3 + 2\cos(\tfrac{3}{4}\pi n - \pi/4) + 11\cos(\tfrac{1}{2}\pi n + \pi/3) \end{aligned}$$

because we can always add a factor of $2\pi n$, which is the source of folding.

Next we want to compute y[n], which is $h[n] * x[n]$. Since we have a sum of sinusoids, *we don't need to do the convolution*; instead, we only need to evaluate the frequency response at each of these frequencies:

At $\hat{\omega} = 0$, $H(e^{j\hat{\omega}}) = \frac{1}{2}\sin(0)e^{j(\pi/2)} = 0$.

At $\hat{\omega} = \frac{3}{4}\pi$, $H(e^{j\hat{\omega}}) = \frac{1}{2}\sin(\frac{3}{2}\pi)e^{j(\pi/2)}e^{-j\frac{3}{2}\pi} = \frac{1}{2}(-1)e^{-j\pi} = \frac{1}{2}$.

At $\hat{\omega} = \frac{1}{2}\pi$, $H(e^{j\hat{\omega}}) = \frac{1}{2}\sin(\pi)e^{-j(\pi/2)}e^{-j2\pi} = 0$.

Therefore, we get

$$y[n] = (\tfrac{1}{2})2\cos(\tfrac{3}{4}\pi n - \pi/4) = \cos(\tfrac{3}{4}\pi n - \pi/4)$$

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