PROBLEM:

Determine the z-transforms of the following sequences:

(a) $x_a[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$.
Express your answer as a polynomial in z^{-1}.

(b) $x_b[n] = u[n] - u[n-5]$.
Express your answer as a ratio of polynomials in z^{-1}.

(c) $x_c[n] = (0.8)^n u[n] + (-0.8)^n u[n]$.
Express your answer as: (1) a sum of rational functions; (2) a ratio of polynomials in z^{-1}; and (3) a product of factors of the form $(1 - az^{-1})$.

(d) $x_d[n] = 2(0.8)^n \cos(0.5\pi n) u[n]$.
Express your answer as: (1) a sum of rational functions; (2) a ratio of polynomials in z^{-1}; and (3) a product of factors of the form $(1 - az^{-1})$.

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SOLUTION

Determine the z-transforms of the following sequences:

(a) $x_a[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$

Use the fact that the transform of $\delta[n-k]$ is z^{-k} to derive

$$X_a(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}$$

(b) $x_b[n] = u[n] - u[n-5]$

Use the fact that the transform of $u[n-k]$ is $z^{-k}/(1-z^{-1})$ to derive

$$X_b(z) = \frac{1}{1-z^{-1}} - \frac{z^{-5}}{1-z^{-1}} = \frac{1-z^{-5}}{1-z^{-1}}$$

Note: you can verify that $x_b[n] = x_a[n]$. Therefore, it must be true that $X_b(z) = X_a(z)$, even though the formulas look different.

(c) $x_c[n] = (0.8)^n u[n] + (-0.8)^n u[n]$.

Express your answer as: (1) a sum of rational functions; (2) a ratio of polynomials in z^{-1}; and (3) a product of factors of the form $(1 - az^{-1})$.

Use the fact that the transform of $a^n u[n]$ is $1/(1-az^{-1})$ to derive form (1):

$$X_c(z) = \frac{1}{1-0.8z^{-1}} + \frac{1}{1+0.8z^{-1}}$$

Then rearrange to get form (2):

$$\begin{aligned} X_c(z) &= \frac{1+0.8z^{-1}}{(1-0.8z^{-1})(1+0.8z^{-1})} + \frac{1-0.8z^{-1}}{(1+0.8z^{-1})(1-0.8z^{-1})} \\ &= \frac{(1+0.8z^{-1}) + (1-0.8z^{-1})}{(1+0.8z^{-1})(1-0.8z^{-1})} \\ &= \frac{2}{(1+0.8z^{-1})(1-0.8z^{-1})} \\ &= \frac{2}{1-0.64z^{-2}} \end{aligned}$$

Then rearrange to get form (3):

$$X_c(z) = 2 \cdot \frac{1}{1+0.8z^{-1}} \cdot \frac{1}{1-0.8z^{-1}}$$

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(d) $x_d[n] = 2(0.8)^n \cos(0.5\pi n)u[n]$.
Express your answer as: (1) a sum of rational functions; (2) a ratio of polynomials in z^{-1}; and (3) a product of factors of the form $(1 - az^{-1})$.

Start by rearraging the sequence into two exponential terms:

$$\begin{aligned} x_d[n] &= 2(0.8)^n \cos(0.5\pi n)u[n] \\ &= 2(0.8)^n \frac{1}{2}\left(e^{j0.5\pi n} + e^{-j0.5\pi n}\right) u[n] \\ &= (0.8e^{j0.5\pi})^n u[n] + (0.8e^{-j0.5\pi})^n u[n] \\ &= (j0.8)^n u[n] + (-j0.8)^n u[n] \end{aligned}$$

then use the fact that the transform of $a^n u[n]$ is $1/(1 - az^{-1})$ to derive form (1):

$$X_d(z) = \frac{1}{1 - j0.8z^{-1}} + \frac{1}{1 + j0.8z^{-1}}$$

Then rearrange to get form (2):

$$\begin{aligned} X_d(z) &= \frac{1 + j0.8z^{-1}}{(1 - j0.8z^{-1})(1 + j0.8z^{-1})} + \frac{1 - j0.8z^{-1}}{(1 + j0.8z^{-1})(1 - j0.8z^{-1})} \\ &= \frac{(1 + j0.8z^{-1}) + (1 - j0.8z^{-1})}{(1 + j0.8z^{-1})(1 - j0.8z^{-1})} \\ &= \frac{2}{(1 + j0.8z^{-1})(1 - j0.8z^{-1})} \\ &= \frac{2}{1 + 0.64z^{-2}} \end{aligned}$$

Then rearrange to get form (3):

$$\begin{aligned} X_d(z) &= 2 \cdot \frac{1}{1 + j0.8z^{-1}} \cdot \frac{1}{1 - j0.8z^{-1}} \\ &= 2 \cdot \frac{1}{1 - 0.8e^{-j0.5\pi}z^{-1}} \cdot \frac{1}{1 - 0.8e^{j0.5\pi}z^{-1}} \end{aligned}$$

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