PROBLEM:

Determine the inverse z-transforms of the following:

(a) $H_a(z) = 1 + 2z^{-2} + 4z^{-4} - 2z^{-6} - z^{-8}$.

(b) $H_b(z) = \dfrac{1 + z^{-2}}{1 - 0.5z^{-1}}$.

(c) $H_c(z) = \dfrac{2}{1 - 0.4z^{-1}} - \dfrac{1}{1 + 0.8z^{-1}}$.

(d) $H_d(z) = \dfrac{1 + 2z^{-1}}{1 + 0.4z^{-1} + 0.32z^{-2}}$.

(e) $H_e(z) = \dfrac{1 + 2z^{-1}}{1 - 0.4z^{-1} + 0.32z^{-2}}$.

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SOLUTION

Determine the inverse z-transforms of the following:

(a) $H_a(z) = 1 + 2z^{-2} + 4z^{-4} - 2z^{-6} - z^{-8}$

Use the fact that the inverse transform of z^{-k} is $\delta[n-k]$ to derive

$$h_a[n] = \delta[n] + 2\delta[n-2] + 4\delta[n-4] - 2\delta[n-6] - \delta[n-8]$$

(b) $H_b(z) = \dfrac{1 + z^{-2}}{1 - 0.5z^{-1}}$

Start by breaking up $H_b(z)$ as follows:

$$H_b(z) = \frac{1}{1 - 0.5z^{-1}} + \frac{z^{-2}}{1 - 0.5z^{-1}}$$

then use the fact that the inverse transform of $z^{-k}/(1 - az^{-1})$ is $a^{n-k}u[n-k]$ to derive

$$h_b[n] = (0.5)^n u[n] + (0.5)^{n-2} u[n-2] = \begin{cases} 0 & n < 0 \\ (0.5)^n & 0 \le n < 2 \\ 5(0.5)^n & n \ge 2 \end{cases}$$

(c) $H_c(z) = \dfrac{2}{1 - 0.4z^{-1}} - \dfrac{1}{1 + 0.8z^{-1}}$

Use the fact that the inverse transform of $1/(1 - az^{-1})$ is $a^n u[n]$ to derive

$$h_c[n] = 2(0.4)^n u[n] - (-0.8)^n u[n] = \left[2(0.4)^n - (-0.8)^n\right] u[n]$$

(d)

$$\begin{aligned} H_d(z) &= \frac{1 + 2z^{-1}}{1 + 0.4z^{-1} - 0.32z^{-2}} \\ &= \frac{1 + 2z^{-1}}{(1 - 0.4z^{-1})(1 + 0.8z^{-1})} \end{aligned}$$

Next use partial fractions to derive the following form:

$$H_d(z) = \frac{A}{1 - 0.4z^{-1}} + \frac{B}{1 + 0.8z^{-1}}$$

which gives the following equation for that equates the numerators of the previous two equations:

$$A(1 + 0.8z^{-1}) + B(1 - 0.4z^{-1}) = 1 + 2z^{-1}$$

By isolating the z^0 and z^{-1} terms, we can derive

$$\begin{aligned} A + B &= 1 \\ 0.8A - 0.4B &= 2 \end{aligned}$$

resulting in $A = 2$ and $B = -1$ and the following equation:

$$H_d(z) = \frac{2}{1 - 0.4z^{-1}} - \frac{1}{1 + 0.8z^{-1}}$$

Thus, $H_d(z) = H_c(z)$ and

$$h_d[n] = h_c[n] = \left[2(0.4)^n - (-0.8)^n\right] u[n]$$

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(e) $H_e(z) = \dfrac{1+2z^{-1}}{1-0.4z^{-1}+0.32z^{-2}}$

The quadratic formula gives the following roots for the denominator:

$$\begin{aligned} z &= 0.2(1 \pm j\sqrt{7}) \\ &= re^{\pm j\hat{\omega}_0} \end{aligned}$$

where $r \approx 0.5657$ and $\hat{\omega}_0 \approx 1.209$, and resulting in the following equation:

$$H_e(z) = \frac{1+2z^{-1}}{(1-re^{j\hat{\omega}_0}z^{-1})(1-re^{-j\hat{\omega}_0}z^{-1})}$$

Next use partial fractions to derive the following form:

$$H_e(z) = \frac{A}{1-re^{j\hat{\omega}_0}z^{-1}} + \frac{B}{1-re^{-j\hat{\omega}_0}z^{-1}}$$

which gives the following equation for that equates the numerators of the previous two equations:

$$A(1-re^{-j\hat{\omega}_0}z^{-1}) + B(1-re^{j\hat{\omega}_0}z^{-1}) = 1+2z^{-1}$$

By isolating the z^0 and z^{-1} terms, we can derive

$$\begin{aligned} A+B &= 1 \\ -Are^{-j\hat{\omega}_0} - Bre^{j\hat{\omega}_0} &= 2 \end{aligned}$$

resulting in $A = be^{-j\phi}$ and $B = be^{j\phi}$ and the following equation:

$$H_e(z) = \frac{be^{-j\phi}}{1-re^{j\hat{\omega}_0}z^{-1}} + \frac{be^{j\phi}}{1-re^{-j\hat{\omega}_0}z^{-1}}$$

where $b \approx 2.138$ and $\phi \approx 1.335$.

Use the fact that the inverse transform of $1/(1-az^{-1})$ is $a^n u[n]$ to derive

$$\begin{aligned} h_e[n] &= be^{-j\phi}r^n e^{j\hat{\omega}_0 n}u[n] + be^{j\phi}r^n e^{-j\hat{\omega}_0 n}u[n] \\ &= br^n\left(e^{-j\phi}e^{j\hat{\omega}_0 n} + e^{j\phi}e^{-j\hat{\omega}_0 n}\right)u[n] \\ &= 2br^n\cos(\hat{\omega}_0 n - \phi)u[n] \end{aligned}$$

where the approximate values of a, b, $\hat{\omega}_0$, and ϕ are given above.

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