PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = 0.8y[n-1] - 0.8x[n] + x[n-1]$$

(a) Determine the system function $H(z)$ for this system. Express $H(z)$ as a ratio of polynomials in z^{-1} and as a ratio of polynomials in z.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.

(d) Show that $|H(e^{j\hat{\omega}})|^2 = 1$ for all $\hat{\omega}$.

(e) If the input to the system is

$$x[n] = 4 + \cos[(\pi/4)n] - 3\cos[(2\pi/3)n]$$

what can you say, without further calculation, about the form of the output $y[n]$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

A linear time-invariant filter is described by the difference equation

$$y[n] = 0.8y[n-1] - 0.8x[n] + x[n-1]$$

(a) Determine the system function $H(z)$ for this system. Express $H(z)$ as a ratio of polynomials in z^{-1} and as a ratio of polynomials in z.

Take the Z transform of the difference equation:

$$Y(z) = 0.8z^{-1}Y(z) - 0.8X(z) + z^{-1}X(z)$$

and rearrange to get the following ratio of polynomials in z^{-1}:

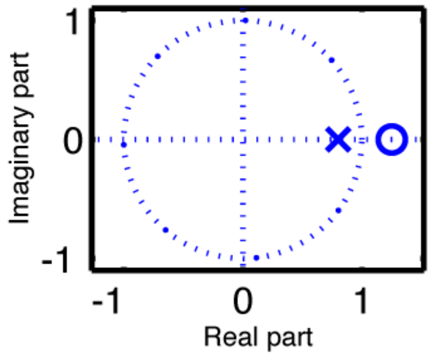
$$H(z) = \frac{Y(z)}{X(z)} = \frac{-0.8 + z^{-1}}{1 - 0.8z^{-1}} = -0.8 \cdot \frac{1 - 1.25z^{-1}}{1 - 0.8z^{-1}}$$

Multiply both numerator and denominator by z to derive

$$H(z) = -0.8 \cdot \frac{z - 1.25}{z - 0.8}$$

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

There is a pole at $z = 0.8$ and a zero at $z = 1.25$.



(c) From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of the system.

$$H(e^{j\hat{\omega}}) = \frac{-0.8 + e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}}$$

(d) Show that $|H(e^{j\hat{\omega}})|^2 = 1$ for all $\hat{\omega}$.

$$\begin{aligned}
|H(e^{j\hat{\omega}})|^2 &= H(e^{j\hat{\omega}})H^*(e^{j\hat{\omega}}) \\
&= \frac{-0.8 + e^{-j\hat{\omega}}}{1 - 0.8e^{-j\hat{\omega}}} \cdot \frac{-0.8 + e^{j\hat{\omega}}}{1 - 0.8e^{j\hat{\omega}}} \\
&= \frac{0.64 - 0.8e^{j\hat{\omega}} - 0.8e^{-j\hat{\omega}} + 1}{1 - 0.8e^{j\hat{\omega}} - 0.8e^{-j\hat{\omega}} + 0.64} \\
&= 1
\end{aligned}$$

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(e) If the input to the system is

$$x[n] = 4 + \cos[(\pi/4)n] - 3\cos[(2\pi/3)n]$$

what can you say, without further calculation, about the form of the output $y[n]$?

Because $|H(e^{j\hat{\omega}})|^2 = 1$, we know that the amplitude of each of the sinusoids (and the value of the constant term) in $y[n]$ is the same as it is in $x[n]$. Thus, we get

$$y[n] = 4 + \cos[(\pi/4)n + \phi_1] - 3\cos[(2\pi/3)n + \phi_2]$$

but the phases ϕ_1 and ϕ_2 are harder to determine. The phase at zero frequency will be zero.

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