PROBLEM:

A linear time-invariant system has system function

$$
\begin{aligned}
H(z) &= \frac{0.0795(1+z^{-1}+z^{-2}+z^{-3})}{1-1.556z^{-1}+1.272z^{-2}-0.398z^{-3}} & (1)\\
&= \frac{0.0795(1+z^{-1})(1-e^{j0.5\pi}z^{-1})(1-e^{-j0.5\pi}z^{-1})}{(1-0.556z^{-1})(1-0.846e^{j0.3\pi}z^{-1})(1-0.846e^{-j0.3\pi}z^{-1})} & (2)\\
&= -0.2+\frac{A}{1-0.556z^{-1}}+\frac{0.17e^{j0.96\pi}}{1-0.846e^{j0.3\pi}z^{-1}}+\frac{0.17e^{-j0.96\pi}}{1-0.846e^{-j0.3\pi}z^{-1}} & (3)
\end{aligned}
$$

(a) Use equation (1) to determine the difference equation that relates the input $x[n]$ to the output $y[n]$ for this system.

(b) Starting with equation(2), use the partial fraction expansion method to show that $A = 0.62$ in equation (3).

(c) Determine the impulse response of this system by finding the inverse z-transform of equation (3). Express your answer in the form

$$h[n] = A_0\delta[n] + A_1 a^n u[n] + A_2 r^n \cos(\hat{\omega}_0 n + \phi)u[n].$$

(d) Use equation (2) to plot the pole and zero locations for this system.

(e) The frequency response of the system can be computed in MATLAB using the following statements:

```
omegahat = -pi:(pi/200):pi;
aa =
bb =
HH=freqz(bb,aa,omegahat);
```

Determine the coefficient vectors `aa` and `bb`.

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SOLUTION

A linear time-invariant system has system function

$$H(z) = \frac{0.0795(1+z^{-1}+z^{-2}+z^{-3})}{1-1.556z^{-1}+1.272z^{-2}-0.398z^{-3}} \qquad (1)$$

$$= \frac{0.0795(1+z^{-1})(1-e^{j0.5\pi}z^{-1})(1-e^{-j0.5\pi}z^{-1})}{(1-0.556z^{-1})(1-0.846e^{j0.3\pi}z^{-1})(1-0.846e^{-j0.3\pi}z^{-1})} \qquad (2)$$

$$= -0.2 + \frac{A}{1-0.556z^{-1}} + \frac{0.17e^{j0.96\pi}}{1-0.846e^{j0.3\pi}z^{-1}} + \frac{0.17e^{-j0.96\pi}}{1-0.846e^{-j0.3\pi}z^{-1}} \qquad (3)$$

(a) Use equation (1) to determine the difference equation that relates the input $x[n]$ to the output $y[n]$ for this system.

Solution is to use (1) and copy the filter coefficients from $H(z)$ into a difference equation:

$$y[n]-1.556y[n-1]+1.272y[n-2]-0.398y[n-3] = 0.0795(x[n]+x[n-1]+x[n-2]+x[n-3]]$$

And it is usually the convention to put $y[n]$ by itself on the left-hand side of the equation:

$$y[n] = 1.556y[n-1]-1.272y[n-2]+0.398y[n-3]+0.0795(x[n]+x[n-1]+x[n-2]+x[n-3]]$$

(b) Starting with equation(2), use the partial fraction expansion method to show that $A = 0.62$ in equation (3).

Solution: Multiply (2) and (3) by $(1-0.556z^{-1})$. In (2), this just cancels the factor $(1-0.556z^{-1})$. In (3), this puts the factor in the numerator of every term except the one with A. Since this expansion must be valid for all values of z, we evaluate these functions at $z = 0.556$. Subsituting this value into MATLAB, we get the result that $A = 0.6172$. We used the following MATLAB commands:

```
aa = 1/0.556
A_numerator = 0.0795*(1+aa)*(1-j*aa)*(1+j*aa)
A_denominator = (1-0.846*exp(j*.3*pi)*aa)*(1-0.846*exp(-j*.3*pi)*aa)
A = A_numerator / A_denominator
```

Another way to do it is to think of the operations needed to put (3) over a common denominator. When you do that, you would cross-multiply by the denominator factors. The result in the numerator would be that the constant term of the numerator is:

$$-0.2 + A + 0.17e^{j0.96\pi} + 0.17e^{-j0.96\pi}$$

which according to (1) would have to be equal to 0.0795. Thus we get

$$A = 0.0795 + 0.2 - 0.17e^{j0.96\pi} - 0.17e^{-j0.96\pi} = 0.2795 - 0.34\cos(0.96\pi) \approx 0.6172$$

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(c) Determine the impulse response of this system by finding the inverse z-transform of equation (3). Express your answer in the form

$$h[n] = A_0\delta[n] + A_1 a^n u[n] + A_2 r^n \cos(\hat{\omega}_0 n + \phi).$$

Solution: Recall that

$$H_1(z) = \frac{1}{1 - az^{-1}} \rightarrow h_1[n] = a^n u[n]$$

Therefore, we get the impulse response as

$$h[n] = -0.2\delta[n] + 0.6172(0.556)^n u[n] + 0.17e^{j0.96\pi}(0.846)^n e^{j0.3\pi n} u[n] + 0.17e^{-j0.96\pi}(0.846)^n e^{-j0.3\pi n} u[n]$$

which can be simplified to:

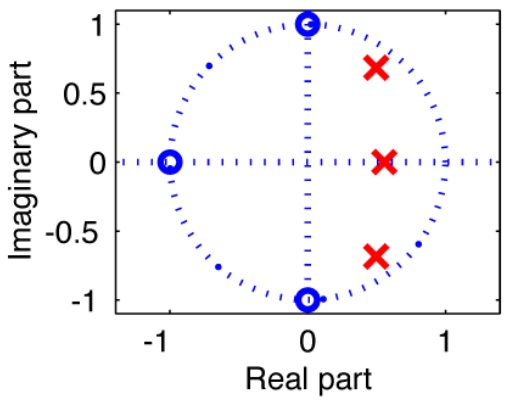
$$h[n] = -0.2\delta[n] + 0.6172(0.556)^n u[n] + 0.34(0.846)^n \cos(0.3\pi n + 0.96\pi)u[n]$$

(d) Use equation (2) to plot the pole and zero locations for this system.

Solution: We have poles at $z = 0.556$, $z = 0.846e^{j0.3\pi}$, and $z = 0.846e^{-j0.3\pi}$.

We have zeros at $z = 1, j, -j$.

We show the plot of these poles and zeros below. We can use
`zplane([-1,j,-j].',[0.556*exp(j*0) 0.846*exp(j*0.3*pi) 0.846*exp(-0.3*j*pi)].')`
to make the plot.



(e) The frequency response of the system can be computed in MATLAB using the following statements:

```
omegahat = -pi:(pi/200):pi;
aa =
bb =
HH = freqz(bb,aa,omegahat);
```

Determine the coefficient vectors aa and bb.

Solution: The MATLAB statements are:

```
b = 0.0795*[1 1 1 1];
a = [1 -1.556 1.272 -0.398];
```

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