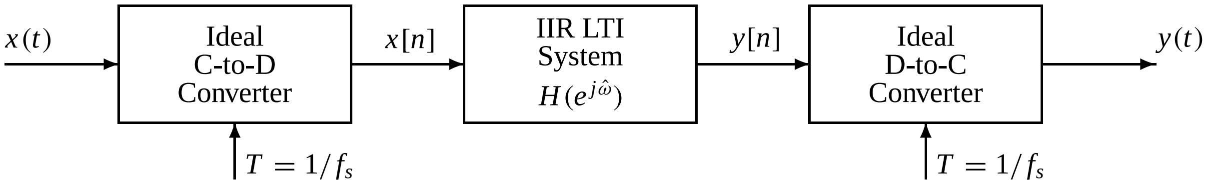
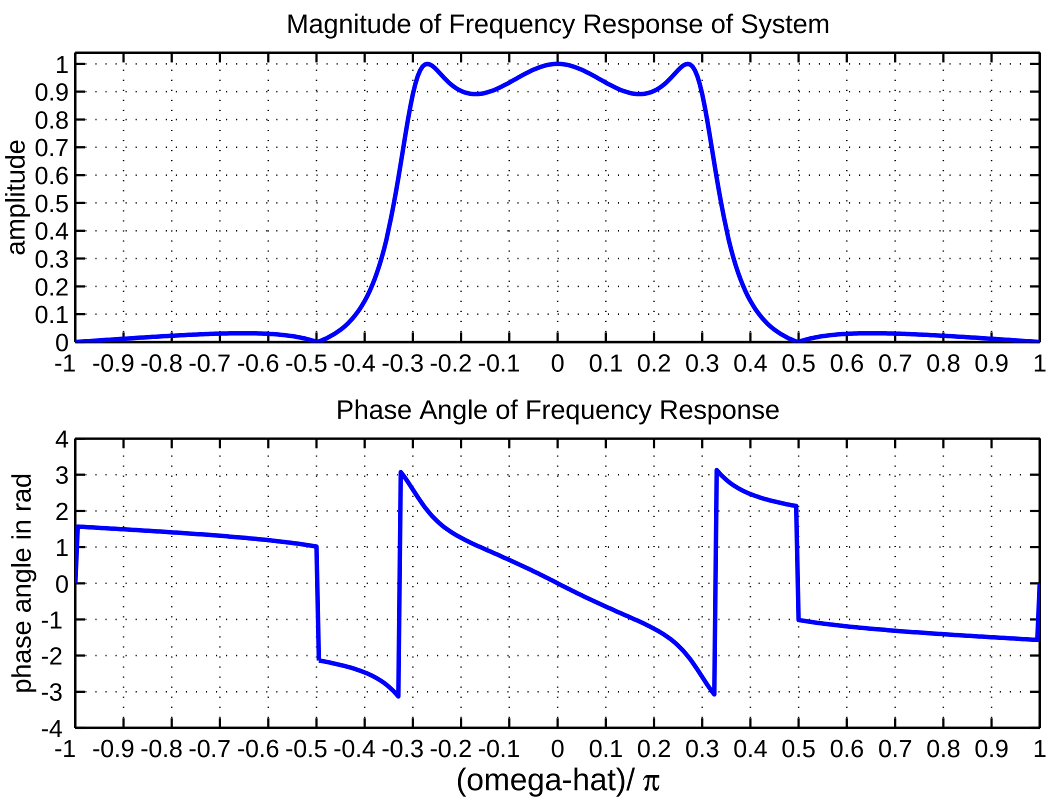
PROBLEM:

Consider the following system for filtering continuous-time signals using an IIR digital filter.



The frequency response of the IIR system, which is the IIR system of Problem 11.1, is plotted in the following figure:



If the sampling rate is $f_s = 10000$ samples/second, and the input to the C-to-D converter is

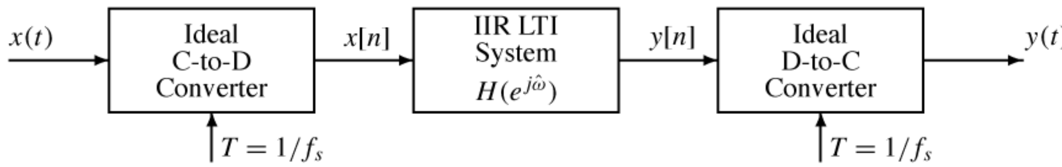
$$x(t) = 10 + 10\cos(2000\pi t) + 10\cos(5000\pi t - 2\pi/3),$$

use the above frequency response plots to determine an expression for $y(t)$, the output of the D-to-C converter.

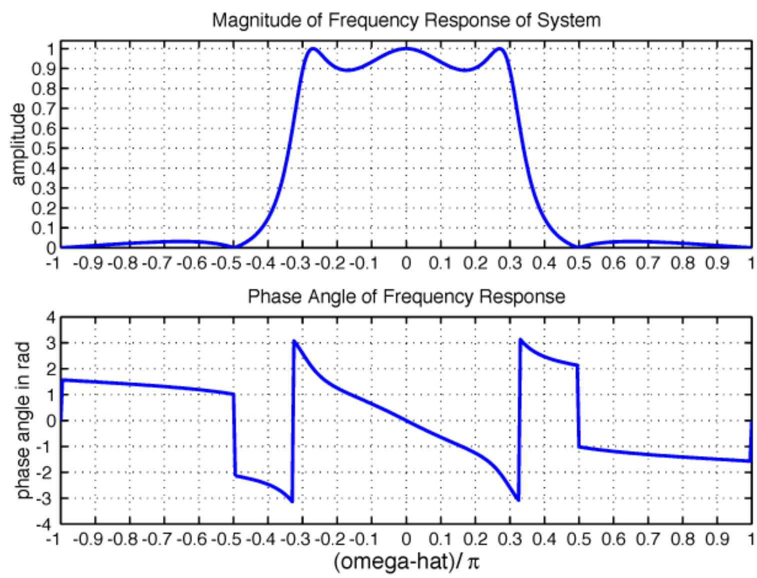
McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Consider the following system for filtering continuous-time signals using an IIR digital filter.



The frequency response of the IIR system, which is the IIR system of Problem 11.1, is plotted in the following figure:



If the sampling rate is $f_s = 10000$ samples/second, and the input to the C-to-D converter is

$$x(t) = 10 + 10\cos(2000\pi t) + 10\cos(5000\pi t - 2\pi/3),$$

use the above frequency response plots to determine an expression for $y(t)$, the output of the D-to-C converter.

Solution: Our sampled signal is represented by

$$x[n] = 10 + 10\cos(0.2\pi n) + 10\cos(0.5\pi n - 2\pi/3)$$

because we convert from analog frequency in rea/sec to digital frequency by dividing by the sampling frequency. Also the is no aliasing to worry about. To find the output response, we need to compute the frequency response at all three frequencies—we can read these values off of the plots.

At $\hat{\omega} = 0.5\pi$, the amplitude is 0. Also, the phase jump of π tells us that we went through a zero.

At $\hat{\omega} = 0$, the amplitude is 1, with zero phase shift.

At $\hat{\omega} = 0.2\pi$, the amplitude is approximately 0.9, and the phase is roughly 1.4 radians.

Therefore, the discrete output is

$$\begin{aligned} y[n] &= 10(1) + 10(0.9)\cos(0.2\pi n + 1.4) + 10(0)\cos(0.5\pi n - 2\pi/3) \\ y[n] &= 10 + 9\cos(0.2\pi n + 1.4) \end{aligned}$$

The output, assuming an ideal discrete to continuous converter is

$$y(t) = 10 + 9\cos(2000\pi t + 1.4)$$

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