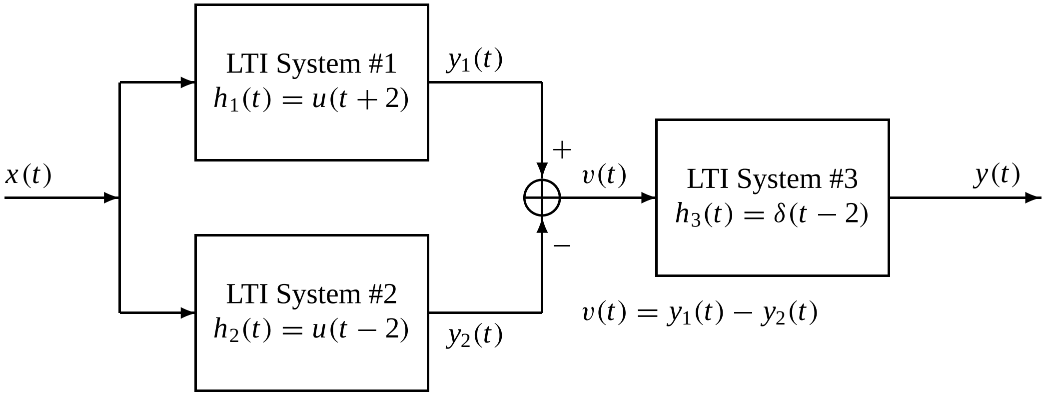
PROBLEM:



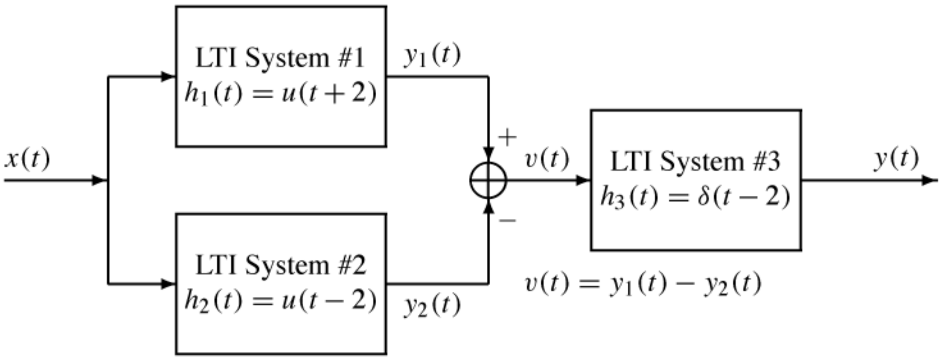
(a) What is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer both as an equation and as a carefully labeled sketch.

(b) Is the overall system a causal system? Explain.
Is it a stable system? Explain.

(c) Are all the subsystems causal? Explain.
Are all the subsystems stable? Explain.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION



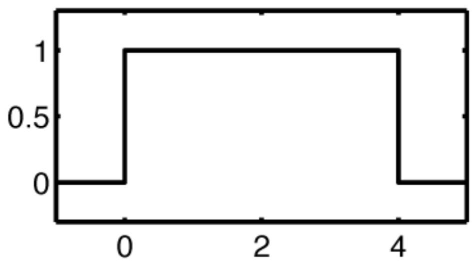
1. What is the impulse response of the overall LTI system (i.e., from $x(t)$ to $y(t)$)? Give your answer <u>both</u> as an equation and as a carefully labeled sketch.

   Solution:

$$\begin{aligned} v(t) &= h_1(t) * x(t) - h_2(t) * x(t) = (h_1(t) + h_2(t)) * x(t) \\ &= (u(t+2) - u(t-2)) * x(t) \end{aligned}$$

$$\begin{aligned} y(t) &= \delta(t-2) * v(t) = \delta(t-2) * (u(t+2) - u(t-2)) * x(t) \\ &= (u(t) - u(t-4)) * x(t) = h(t) * x(t) \end{aligned}$$

   Therefore, $h(t) = u(t) - u(t-4)$



   MATLAB code to generate this plot: `plot([-1 0 0 4 4 5],[0 0 1 1 0 0]);`

2. Is the overall system a causal system? (Explain to receive credit.) Is it a stable system? (Explain to receive credit.)

   Solution: This solution is causal, since the impulse response, $h(t) = u(t) - u(t-4)$, is zero for $t < 0$. This solution is stable, since the integral over $|h(t)|$ for all t is finite:

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_0^4 1\, dt = 4 < \infty$$

3. Are all the subsystems causal? (Explain to receive credit.) Are all the subsystems stable? (Explain to receive credit.)

   Solution:

   $h_1(t)$ is not causal, because its impulse response starts before t=0.

   $h_1(t)$ and $h_2(t)$ are not stable because the integral over all time of a unit-step signal is not finite. For example,

$$\int_{-\infty}^{\infty} |h_1(t)| dt = \int_{-2}^{\infty} 1\, dt \rightarrow \infty$$

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