PROBLEM:

Use the delay property of Fourier transforms,

$$x(t-t_d) \Longleftrightarrow e^{-j\omega t_d} X(j\omega),$$

to determine the Fourier transforms of the following signals:

(a) $x(t) = \delta(t-5)$

(b) $x(t) = 20\dfrac{\sin(200\pi(t-10))}{\pi(t-10)}$

(c) $x(t) = e^{-4t}u(t) - e^{-4t}u(t-10) = e^{-4t}u(t) - e^{-40}e^{-4(t-10)}u(t-10)$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Use the delay property of Fourier transforms,

$$x(t-t_d) \Longleftrightarrow e^{-j\omega t_d} X(j\omega),$$

to determine the Fourier transforms of the following signals:

(a) $x(t) = \delta(t-5)$

Given the fact that the Fourier transform of $\delta(t)$ is equal to 1, the Fourier transform of this delayed impulse is $X(j\omega) = e^{-j5\omega}$.

(b) $x(t) = 20\dfrac{\sin(200\pi(t-10))}{\pi(t-10)}$

The Fourier transform of $\sin(200\pi t)/\pi t$ is equal to the box defined by difference of step functions: $u(\omega+200\pi) - u(\omega-200\pi)$. $x(t)$ is equal to this function scaled (in amplitude) by a factor of 20 and delayed by 10 seconds. Thus, the Fourier transform of $x(t)$ is

$$X(j\omega) = 20e^{-j10\omega}[u(\omega+200\pi) - u(\omega-200\pi)]$$

(c) $x(t) = e^{-4t}u(t) - e^{-4t}u(t-10) = e^{-4t}u(t) - e^{-40}e^{-4(t-10)}u(t-10)$

The Fourier transform of $e^{-4t}u(t)$ is $1/(4+j\omega)$. Thus, the Fourier transform of $x(t)$ is

$$\begin{aligned} X(j\omega) &= \frac{1}{4+j\omega} - e^{-40}e^{-j10\omega}\frac{1}{4+j\omega} \\ &= \frac{1-e^{-j10\omega-40}}{4+j\omega} \end{aligned}$$

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