PROBLEM:

The input to an LTI system is $x(t) = 10 + 2\delta(t-2) + \dfrac{\sin(2000\pi t)}{\pi t}$

and the frequency response of the system is $H(j\omega) = \begin{cases} 10 & |\omega| < 1000\pi \\ 0 & |\omega| > 1000\pi. \end{cases}$

(a) Determine $X(j\omega)$, the Fourier transform of $x(t)$. (You may give part of your answer as a sketch if that is most convenient.)

(b) Using the specific $X(j\omega)$ determined in (a), obtain a simple expression for $Y(j\omega)$, the Fourier transform of the output of the LTI system, in terms of $H(j\omega)$.

(c) Use the inverse Fourier transform to determine $y(t)$, the output of the system for input $x(t)$. *Do not leave your answer in terms of* $h(t)$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

(a) For the input signal,

$$x(t) = 10 + 2\delta(t-2) + \frac{\sin(2000\pi t)}{\pi t},$$

the Fourier transform can be done one term at a time:

$$X(j\omega) = 20\pi\delta(\omega) + 2e^{-j2\omega} + u(\omega + 2000\pi) - u(\omega - 2000\pi)$$

Note that the ideal LPF in frequency is represented by

$$u(\omega + 2000\pi) - u(\omega - 2000\pi) = \begin{cases} 1 & \text{for } |\omega| < 2000\pi \\ 0 & \text{for } |\omega| > 2000\pi \end{cases}$$

(b) We get the Fourier transform of the output by passing each term thru $H(j\omega)$ which has a cutoff frequency at $\omega_c = 1000\pi$.

$$\begin{aligned}
Y_1(j\omega) &= 20\pi\delta(\omega)H(j\omega) = 20\pi\delta(\omega)H(j0) = 200\pi\delta(\omega) \\
Y_2(j\omega) &= 2e^{-j2\omega}H(j\omega) = 2e^{-j2\omega}[10u(\omega + 1000\pi) - 10u(\omega - 1000\pi)] \\
Y_3(j\omega) &= [u(\omega + 2000\pi) - u(\omega - 2000\pi)]\,H(j\omega) \\
&= [u(\omega + 2000\pi) - u(\omega - 2000\pi)]\,[10u(\omega + 1000\pi) - 10u(\omega - 1000\pi)] \\
&= 10[u(\omega + 1000\pi) - u(\omega - 1000\pi)]
\end{aligned}$$

Adding these together gives the final result:

$$Y(j\omega) = 200\pi\delta(\omega) + (20e^{-j2\omega} + 10)[u(\omega + 1000\pi) - u(\omega - 1000\pi)]$$

(c) We can get y(t) by the inverse Fourier transform of the 3 terms in the equation above.

$$y(t) = 100 + 20\frac{\sin(1000\pi(t-2))}{\pi(t-2)} + 10\frac{\sin(1000\pi t)}{\pi t}$$

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