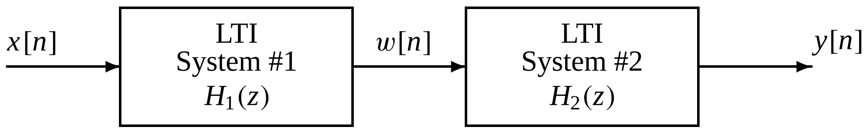
PROBLEM:

Consider the following cascade system:

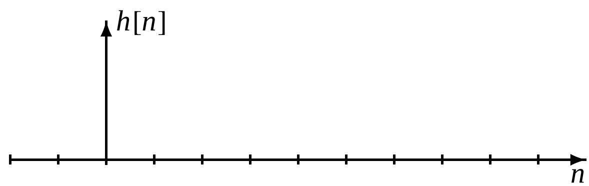


where

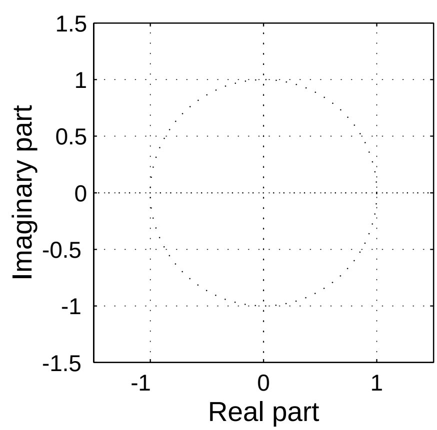
$$H_1(z) = 2 + 2z^{-2} \quad \text{and} \quad H_2(z) = 1 + \tfrac{1}{2}z^{-1}.$$

(a) Determine the system function $H(z)$ of the overall system. Express your answer as a polynomial in z^{-1}.

(b) Determine and plot the impulse response $h[n]$ of the overall system.



(c) Return to your result in part (a). Express $H(z)$ as the product of a constant and three first-order factors each written in the form $(1 - az^{-1})$. From this, determine the zeros and poles of $H(z)$ and plot them in the z-plane plot below.

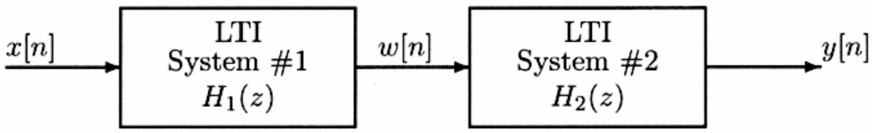


(d) If the input is $x[n] = Ae^{j\phi}e^{j\hat{\omega}_0 n}$ for $-\infty < n < \infty$, for what values of $-\pi \le \hat{\omega}_0 \le \pi$ will the output be $y[n] = 0$ for $-\infty < n < \infty$?

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

Consider the following cascade system:



where

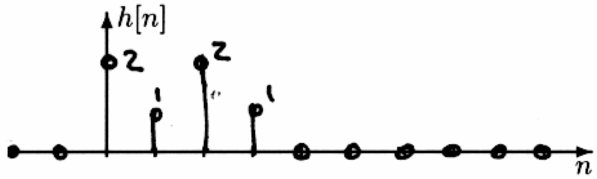
$$H_1(z) = 2 + 2z^{-2} \quad \text{and} \quad H_2(z) = 1 + \tfrac{1}{2}z^{-1}.$$

(a) Determine the system function $H(z)$ of the overall system. Express your answer as a polynomial in z^{-1}.

$$H(z) = H_1(z)\,H_2(z) = (2+2z^{-2})(1+\tfrac{1}{2}z^{-1})$$
$$= 2 + z^{-1} + 2z^{-2} + z^{-3}$$

(b) Determine and plot the impulse response $h[n]$ of the overall system.

$$h[n] = 2\delta[n] + \delta[n-1]$$
$$+ 2\delta[n-2] + \delta[n-3]$$



(c) Return to your result in part (a). Express $H(z)$ as the product of a constant and three first-order factors each written in the form $(1 - az^{-1})$. From this, determine the zeros and poles of $H(z)$ and plot them in the z-plane plot below.

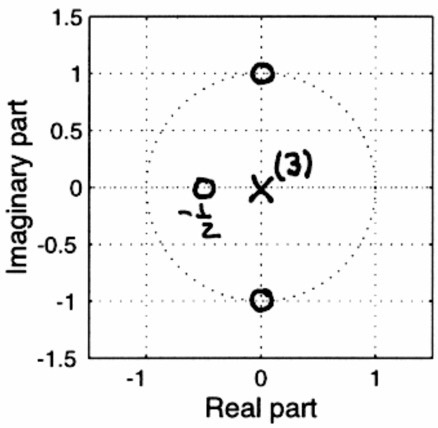
Zeros satisfy $2+2z^{-2} = 0$

$2(z^2+1)/z^2 = 0 \Rightarrow z = \pm j$

and $1+\frac{1}{2}z^{-1} = \frac{z+1/2}{z} \Rightarrow z = -\frac{1}{2}$

There are 3 poles at $z=0$

$$H(z) = 2(1-e^{j\pi/2}z^{-1})(1-e^{-j\pi/2}z^{-1})$$
$$\cdot(1+\tfrac{1}{2}z^{-1})$$



(d) If the input is $x[n] = Ae^{j\phi}e^{j\hat{\omega}_0 n}$ for $-\infty < n < \infty$, for what values of $-\pi \le \hat{\omega}_0 \le \pi$ will the output be $y[n] = 0$ for $-\infty < n < \infty$?

$y[n] = 0$ for all n if $\omega_0 = \pm\frac{\pi}{2}$

since $H(e^{\pm j\pi/2}) = 0$

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