PROBLEM:

The system function of a discrete-time LTI system has the following equivalent forms:

$$H(z) = \frac{2 + 2z^{-1}}{1 - 0.25z^{-2}} = \frac{2 + 2z^{-1}}{(1 - 0.5z^{-1})(1 + 0.5z^{-1})} = \frac{3}{1 - 0.5z^{-1}} - \frac{1}{1 + 0.5z^{-1}}$$

(a) Determine the impulse response of this system; i.e., determine the output $h[n]$ when the input is $\delta[n]$.

(b) Using the form

$$H(z) = \frac{2 + 2z^{-1}}{1 - 0.25z^{-2}},$$

determine an expression for the frequency response as a function of $\hat{\omega}$.

(c) Use the frequency response function to determine the output $y[n]$ when the input is

$$x[n] = e^{j(\pi/2)n} \quad \text{for } -\infty < n < \infty$$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

The system function of a discrete-time LTI system has the following equivalent forms:

$$H(z) = \frac{2+2z^{-1}}{1-0.25z^{-2}} = \frac{2+2z^{-1}}{(1-0.5z^{-1})(1+0.5z^{-1})} = \frac{3}{1-0.5z^{-1}} - \frac{1}{1+0.5z^{-1}}$$

(a) Determine the impulse response of this system; i.e., determine the output $h[n]$ when the input is $\delta[n]$.

Use the partial fraction form and

$$a^n u[n] \Longleftrightarrow \frac{1}{1-az^{-1}}$$

$$h[n] = 3(0.5)^n u[n] - (-0.5)^n u[n]$$

(b) Using the form

$$H(z) = \frac{2+2z^{-1}}{1-0.25z^{-2}},$$

determine an expression for the frequency response as a function of $\hat{\omega}$.

Substitute $z = e^{j\hat{\omega}}$

$$H(e^{j\hat{\omega}}) = \frac{2 + 2e^{-j\hat{\omega}}}{1 - 0.25e^{-j\hat{\omega}2}}$$

(c) Use the frequency response function to determine the output $y[n]$ when the input is

$$x[n] = e^{j(\pi/2)n} \quad \text{for } -\infty < n < \infty$$

$$y[n] = H(e^{j\pi/2})e^{j(\pi/2)n}$$

$$= \frac{2 + 2e^{-j\pi/2}}{1 - 0.25e^{-j\pi}} e^{j\frac{\pi}{2}n} = \frac{2-2j}{1.25} e^{j\frac{\pi}{2}n}$$

$$= \frac{2\sqrt{2}}{1.25} e^{-j\pi/4} e^{j\frac{\pi}{2}n} = \frac{2\sqrt{2}}{1.25} e^{j\left(\frac{\pi}{2}n - \pi/4\right)}$$