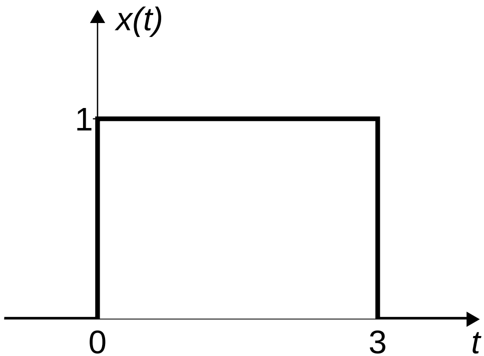
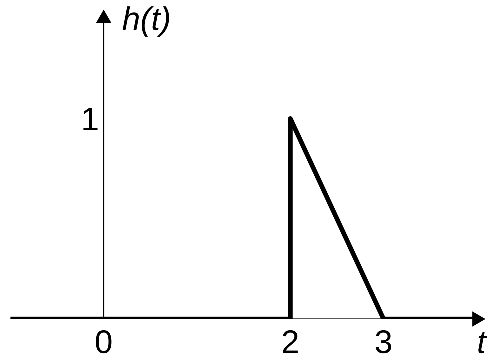
PROBLEM:

The following figure shows the signal $x(t) = u(t) - u(t-3)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right below) is the triangular function

$$h(t) = \begin{cases} 3 - t & 2 < t < 3 \\ 0 & \text{otherwise.} \end{cases}$$

The output of the LTI system is $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$.





(a) Sketch $h(3-\tau)$ as a function of τ in the space below.

(b) For what values of t can you state with certainty that $y(t) = 0$? **Draw appropriate sketches of $x(\tau)$ and $h(t-\tau)$ to aid your solution.**

(c) Determine the value of $y(t)$ at $t = 3$; that is, determine $y(3)$. **Note carefully: You do not need to evaluate $y(t)$ for all t, only for $t = 3$, and you will not need to "do" any integrals.**

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

The following figure shows the signal $x(t) = u(t) - u(t-3)$, which is the input to a continuous-time LTI system whose impulse response (shown on the right below) is the triangular function

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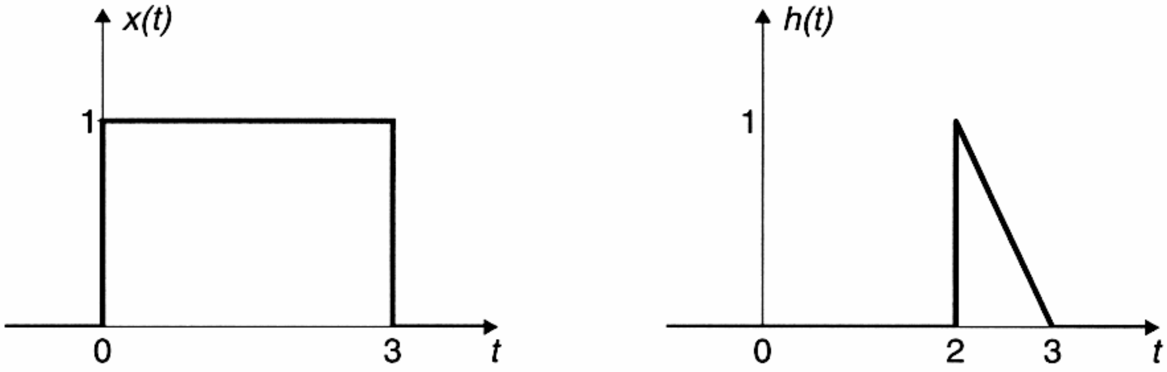
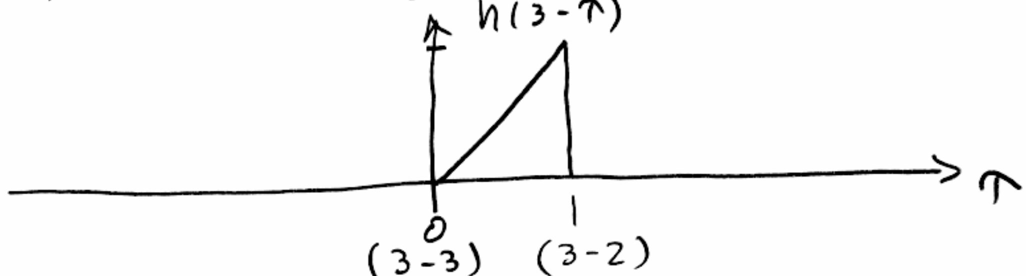


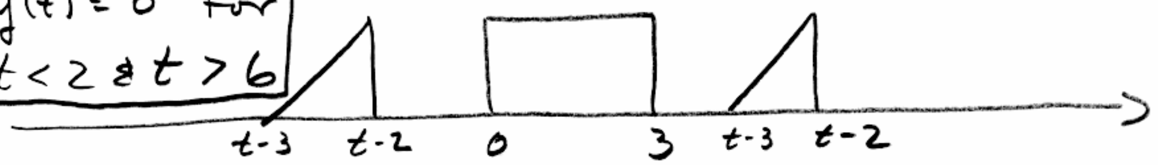
(a) Sketch $h(3-\tau)$ as a function of τ in the space below.



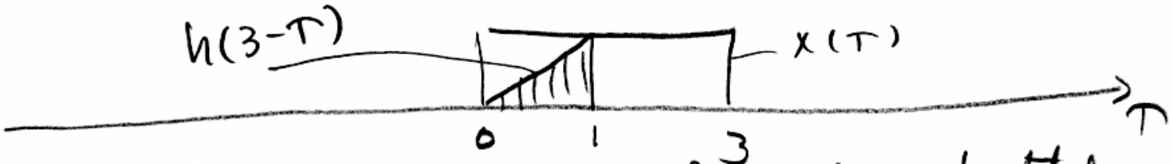
(b) For what values of t can you state with certainty that $y(t) = 0$? **Draw appropriate sketches of $x(\tau)$ and $h(t-\tau)$ to aid your solution.**

$t-2<0 = t<2$ $\qquad$ $t-3>3 \Rightarrow t>6$

$\boxed{y(t) = 0 \text{ for } t<2 \;\&\; t>6}$



(c) Determine the value of $y(t)$ at $t=3$; that is, determine $y(3)$. **Note carefully: You do not need to evaluate $y(t)$ for all t, only for $t=3$, and you will not need to "do" any integrals.** To determine $y(3)$, plot $x(\tau)$ & $h(3-\tau)$



$y(3)$ is the area of the flipped & shifted triangle so $y(3) = 1/2$

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