



PROBLEM:

Simplify the following complex-valued expressions. Give your answer in either rectangular or polar form, whichever is most convenient. In parts (a)-(d) assume that A , α , and ϕ are positive real numbers. Your answers to parts (a)-(d) will be in terms of these quantities.

- (a) For $z = Ae^{-j\pi/3}$, determine a simple expression for $\Im\{z^*\}$.
- (b) For $z = Ae^{-j\pi/3}$, determine a simple expression for $z + z^*$.
- (c) For $z = 10e^{j\phi}$, determine a simple expression for $\Re\{jz\}$.
- (d) For $z = -\alpha + j\alpha$, determine a simple expression for z in polar form.



$$a) \quad z = A e^{-j\frac{\pi}{3}}$$

$$z^* = A e^{+j\frac{\pi}{3}}$$

$$\text{Im}(z^*) = A \sin\left(\frac{\pi}{3}\right) = A \frac{\sqrt{3}}{2}$$

$$b) \quad z = A e^{-j\frac{\pi}{3}}$$

NOTE: $z + z^* = 2 \text{Re}(z)$

$$z + z^* = 2A \cos\left(\frac{\pi}{3}\right) = A$$

$$c) \quad z = 10 e^{j\phi}$$

$$jz = 10 e^{j\left(\phi + \frac{\pi}{2}\right)}$$

$$\text{Re}(jz) = 10 \cos\left(\phi + \frac{\pi}{2}\right)$$

$$= -10 \sin(\phi)$$

NOTE: $-\sin(\theta) = \cos\left(\theta + \frac{\pi}{2}\right)$

$$d) \quad z = -\alpha + j\alpha$$

$$= \alpha(-1 + j)$$

$$= \sqrt{2}\alpha e^{j\frac{3}{4}\pi}$$

