PROBLEM:

A periodic signal is represented by the Fourier Series synthesis formula:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j2400\pi kt} \qquad \text{where} \quad a_k = \begin{cases} \dfrac{1}{4 + j2k} & \text{for } k = -3, -2, -1, 0, 1, 2, 3 \\ 0 & \text{for } |k| > 3 \end{cases}$$

(a) Determine a formula for the signal $x(t)$ and a sum of sinusoids, using the cosine form.

(b) Determine the minimum sampling rate f_s (in Hz) such that $x(t)$ can be reconstructed from its samples, $x(n/f_s)$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

Note that $a_{-k} = a_k^*$, hence $x(t)$ is a REAL signal

Combine the terms for $k=-3$ and $k=3$, $k=-2$ and $k=2$, $k=-1$ and $k=1$. For $k=0$, the DC term is $\frac{1}{4}$.

i) Here we go:

$$a_1 e^{j2400\pi t} + a_{-1} e^{-j2400\pi t}$$

$$a_1 = \frac{1}{4+2j} = 0.2 - 0.1j = 0.2236\, e^{j\tan^{-1}(-0.5)} = 0.2236\, e^{-j\pi/6}$$

<u>Remark!</u> $\tan^{-1}(-0.5)$ does <u>not really</u> define the argument (phase angle) φ_1) uniquely.

Correct is:

$$\left\{\begin{array}{l} \sin\varphi_1 = \frac{-0.1}{0.2236} \\ \cos\varphi_1 = \frac{0.2}{0.2236} \end{array}\right. \rightarrow \varphi_1 = -\pi/6$$

Likewise:

$$a_2 = \frac{1}{4+4j} = 0.125 - 0.125j = 0.1768\, e^{j\tan^{-1}(-1)} = 0.1768\, e^{-j\pi/4}$$

(same caveat)

$$a_3 = \frac{1}{4+6j} = 0.0769 - 0.1154j = 0.1387\, e^{j\tan^{-1}(-1.5)} = 0.1387\, e^{-j\,0.9828}$$

(same caveat)

Noting that $\frac{X_k}{2} = a_k \Rightarrow$

$$\boxed{\begin{array}{l} x(t) = 0.25 + 0.4472\cos(2400\pi t - 0.4636) + \\ \quad + 0.3536\cos(4800\pi t - 0.7854) + \\ \quad + 0.2774\cos(7200\pi t - 0.9828) \end{array}}$$

ii) The highest frequency is $\frac{7200\pi}{2\pi} = 3600\,\text{Hz}$.

By Nyquist's results, the minimum sampling rate is $3600 \times 2 = \underline{\underline{7200\,\text{Hz}}}$