PROBLEM:

In the rotating disk and strobe demo described in Chapter 4 of *SP-First*, we observed that different flashing rates of the strobe light would make the spot on the disk stand still.

(a) Assume that the disk is rotating in the counter-clockwise direction at a constant speed of 12 revolutions per second. Express the movement of the spot on the disk as a rotating complex exponential $e^{j\omega t}$ (a.k.a. a rotating phasor).

(b) If the strobe light can be flashed at a rate of n flashes *per second* where n is an integer greater than zero, determine all possible flashing rates such that the disk can be made to stand still.
NOTE: the only possible flashing rates are integers: 1 per second, 2 per second, 3 per second, etc.

(c) Now assume that the flashing rate is fixed, so that the interval between flashes is 100 millisec. Explain how the spot will move and write a complex phasor that gives the position of the spot at each flash.

(d) Draw a spectrum plot of the discrete-time signal in part (c) to explain your answer.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458. © 2003 Pearson Education, Inc.

SOLUTION

a) For 12 rpm, counterclockwise, the phasor (with reference frequency 0) is $e^{j\,12\times 2\pi t} = e^{j24\pi t}$

(taking its magnitude (distance from axis of rotation) to be one)

b) For n flashes per second, the time between successive flashes is $\Delta t = \frac{1}{n}$ $(= T_s)$ Hence for apparent standstill an integer number of revolutions between successive flashes is needed, or $\frac{12}{n}$ must be an integer $\Rightarrow n = 1, 2, 3, 4, 6, 12.$

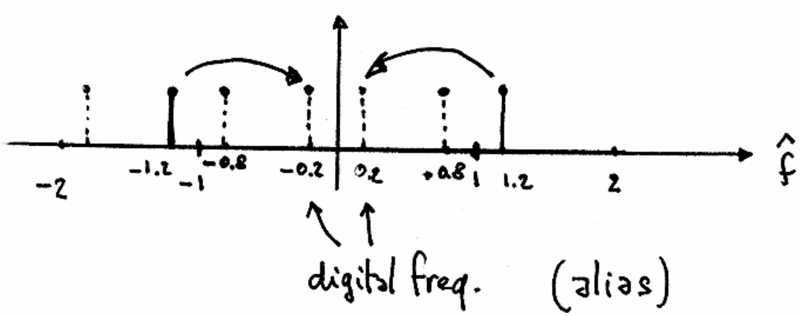
How does this problem relate to the course material on sampling?

$$\hat{\omega} = (24\pi).T_s = 2\pi \cdot \frac{12}{n} \Rightarrow \frac{12}{n} \text{ integer.}$$

c) $T_s = 100\text{ msec} \Rightarrow x[k] = e^{j24\pi.(k0.1)} = e^{j2.4k\pi} = e^{j0.4\pi k}$

d) $f_0 = 12$ $\rightarrow$ normalized frequency $= 1.2 = \hat{f}_0$

$f_s = 10$ or $\hat{\omega} = 2.4\pi\text{rad}$



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