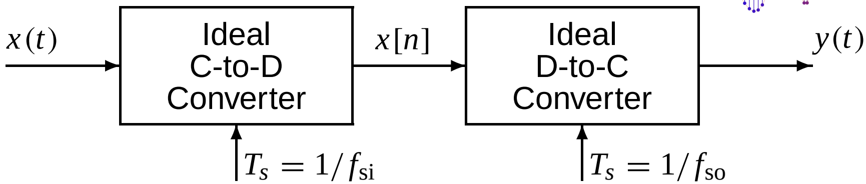
PROBLEM:

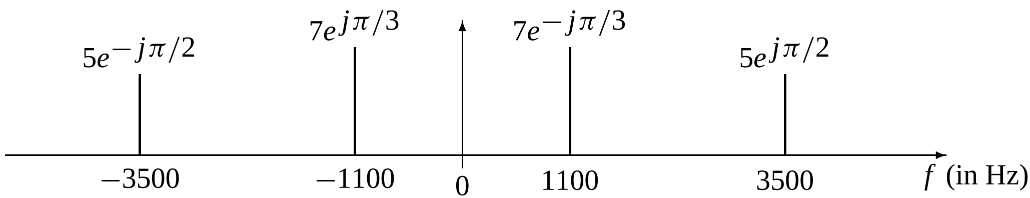


(a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 10\cos(0.25\pi n + 3\pi/4)$$

If the sampling rate of the C-to-D converter is $f_{\text{si}} = 4000$ samples/second, many *different* continuous-time signals $x(t) = x_\ell(t)$ could have been inputs to the above system. Determine two such inputs with frequency less than 4000 Hz; i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_{\text{si}}) = x_2(nT_{\text{si}})$ if $T_{\text{si}} = 1/4000$.

(b) If the input $x(t)$ is given by the two-sided spectrum representation shown below,



Determine the *discrete-time* spectrum for $x[n]$ when $f_{\text{si}} = 4000$ samples/sec. Make a $\boxed{\text{plot}}$ for your answer, but label the frequency, amplitude and phase of each spectral component.

(c) Using the discrete-time spectrum from part (b), determine the analog frequency components in the output $y(t)$ when the sampling rate of the D-to-C converter is $f_{\text{so}} = 8000$ Hz. In other words, the sampling rates of the two converters are different.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Upper Saddle River, NJ 07458.

SOLUTION

a) $10\cos(0.25\pi n + 3\pi/4) = 10\cos\left(2\pi \cdot \frac{500n}{4000} + 3\pi/4\right)$

$$= 10\cos\left(2\pi \cdot 500\, nT_{si} + 3\pi/4\right)$$

$$= 10\cos\left(2\pi \cdot 500\, t + 3\pi/4\right)\Big|_{t=nT_{si}}$$

$$\boxed{\rightarrow\ x_1(t) = 10\cos\left(2\pi \cdot 500t + 3\pi/4\right)}$$

Also

$$10\cos(0.25\pi n + 3\pi/4) = 10\cos(-0.25\pi n - 3\pi/4)$$

$$= 10\cos\left[(2\pi - 0.25\pi)n - 3\pi/4\right]$$

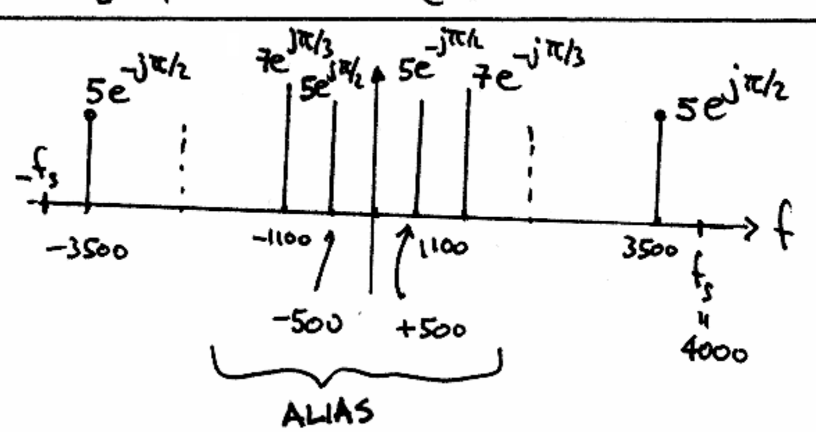
$$= 10\cos\left[2\pi \cdot \frac{3500}{4000} n - 3\pi/4\right]$$

$$= 10\cos\left[2\pi \cdot 3500\, nT_{si} - 3\pi/4\right]$$

$$= 10\cos\left[2\pi \cdot 3500\, t - 3\pi/4\right]\Big|_{t=nT_{si}}$$

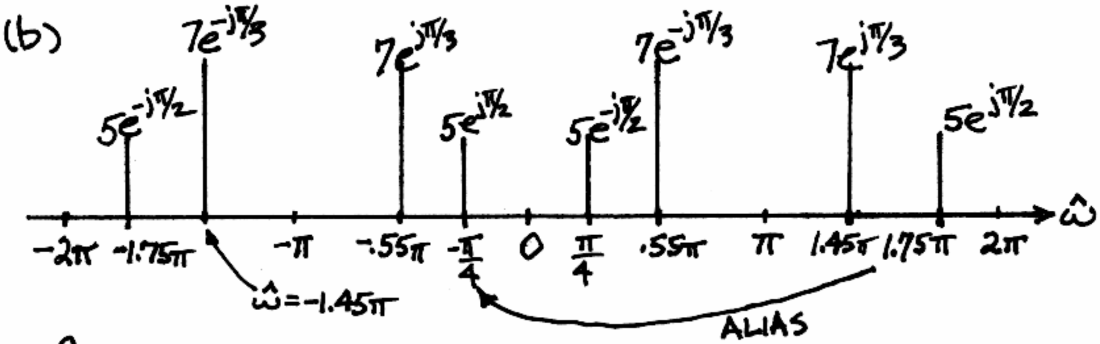
$$\boxed{\rightarrow\ x_2(t) = 10\cos\left(2\pi \cdot 3500t - 3\pi/4\right)}$$

b)



discrete spectrum.

(b)



$f_s = 4000$ Hz

$f = 1100$ Hz $\Rightarrow$ $\hat{\omega} = 2\pi\left(\frac{1100}{4000}\right) = 0.55\pi$ rads.

$f = 3500$ Hz $\Rightarrow$ $\hat{\omega} = 2\pi\left(\frac{3500}{4000}\right) = 1.75\pi$ rads.

$\hat{\omega} = 1.75\pi$ rads. is EQUIVALENT to $\hat{\omega} = 1.75\pi - 2\pi = -0.25\pi$ rads

Also $\hat{\omega} = 0.55\pi$ rads is EQUIV. to $\hat{\omega} = 0.55\pi - 2\pi = -1.45\pi$ rads.

In the plot above, the range $-2\pi < \hat{\omega} < 2\pi$ is shown. This portrays the fact that the spectrum for the discrete-time signal is periodic with period $= 2\pi$

c) The converter D→C is twice as fast. This doubles all frequencies in the reconstruction.

$\therefore$ analog frequency components 1000 Hz
2200 Hz.

McClellan, Schafer, and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

(c) The ideal D-C converter maps an $\hat{\omega}$ frequency back to f (Hz) via:

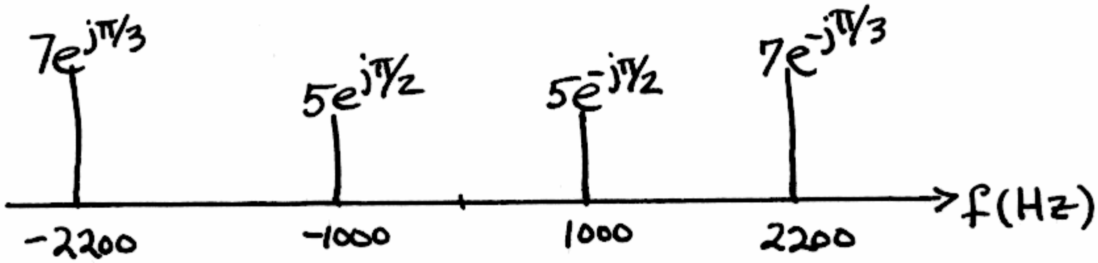
$$\hat{\omega} = 2\pi(f/f_s) \Longrightarrow f = \left(\frac{\hat{\omega}}{2\pi}\right) f_s$$

It also uses the lowest freqs, so it takes everything between $-\pi$ & π.

$$\hat{\omega} = \pm\pi/4 \longrightarrow f = \left(\frac{\pm\pi/4}{2\pi}\right) 8000 = \pm 1000 \text{ Hz}$$

$$\hat{\omega} = \pm 0.55\pi \longrightarrow f = \left(\frac{\pm 0.55\pi}{2\pi}\right) 8000 = \pm 2200 \text{ Hz}$$

The spectrum for the analog signal at the output of the C-D converter is:



The formula for the output signal is:

$$y(t) = 10\cos(2\pi(1000)t - \pi/2) + 14\cos(2\pi(2200)t - \pi/3)$$

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